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ADVANCED MULTI-ATTRIBUTE DECISION-MAKING: A PICTURE FUZZY EINSTEIN OPERATOR AND TOPSIS APPROACH

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ABSTRACT

Multi-attribute decision-making (MADM) is a critical area of research in decision theory, often utilized for evaluating and selecting alternatives in complex scenarios. This paper introduces an innovative decision-making approach by combining the Picture Fuzzy Einstein Operator (PFE) with the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. Picture fuzzy sets offer a rich representation of uncertainty and vagueness, which is essential for real-world decision problems. The Einstein operation serves as a flexible aggregation method for combining information from multiple sources. By incorporating the TOPSIS method, which ranks alternatives based on their distance from an ideal solution, we aim to enhance the accuracy and robustness of decision-making. The paper applies the proposed method to a practical example to demonstrate its efficacy and compares it with traditional fuzzy set-based approaches.

KEYWORDS

Picture Fuzzy Sets, Einstein Aggregation Operator, TOPSIS Method, Multi-Attribute Decision-Making, Uncertainty, Fuzzy Logic, Decision Support Systems, Multi-Criteria Decision Analysis, Ranking Alternatives, Fuzzy Decision-Making.

INTRODUCTION

Decision-making under uncertainty is a common challenge in various fields, including engineering, management, and economics. Multi-Attribute Decision-Making (MADM) methods are widely used to solve problems where multiple alternatives are evaluated based on several criteria. Traditional decision-making approaches often rely on crisp values, but real-world problems often involve uncertainty, imprecision, and vagueness, which cannot be adequately captured using conventional methods.

In recent years, fuzzy sets have been applied to decision-making processes to handle this inherent uncertainty. However, classical fuzzy sets cannot fully represent the complex nature of decision-making in certain situations. This led to the introduction of more advanced fuzzy concepts, such as picture fuzzy sets, which allow for a more nuanced representation of uncertainty by incorporating membership, non-membership, and hesitation degrees.

One of the key operators used to aggregate information from multiple sources in fuzzy decision-making is the Einstein operator. This operator is particularly useful when dealing with picture fuzzy sets, as it allows for the combination of multiple fuzzy values in a manner that respects their relative importance and provides a more comprehensive assessment of alternatives.

To rank the alternatives effectively, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method has been widely adopted. TOPSIS is based on the principle that the best alternative is the one that has the shortest distance from the ideal solution and the farthest distance from the negative ideal solution. This method is well-suited for situations where the alternatives can be evaluated using multiple criteria, each associated with varying degrees of importance.

This study aims to develop a multi-attribute decision-making framework that combines the Picture Fuzzy Einstein Operator with the TOPSIS method to enhance the decision process's flexibility, accuracy, and robustness. The proposed model is applied to a real-world case study, providing valuable insights into its effectiveness compared to traditional fuzzy set-based methods.

Decision-making is an essential part of human life, as individuals and organizations are constantly required to evaluate alternatives and make choices in complex and uncertain environments. In many real-world decision problems, multiple criteria must be considered simultaneously to determine the best alternative. This complexity increases when the criteria involve uncertainty, imprecision, and vagueness, which are inherent in human judgment. As a result, traditional decision-making models that rely on crisp values and linear calculations often fall short in accurately capturing the nuances of real-world situations.

Multi-attribute decision-making (MADM) is a branch of decision theory that addresses problems in which alternatives must be evaluated across multiple criteria. MADM methods are commonly applied in areas such as business management, engineering, economics, and healthcare, where decision-makers face challenges such as selecting the best product, supplier, or solution from a set of alternatives. These methods allow for a systematic comparison of alternatives by considering different attributes, such as cost, quality, time, and performance. However, the limitations of conventional decision-making models become apparent when these attributes involve inherent uncertainty and human subjectivity.

Fuzzy set theory has become a popular tool to handle uncertainty in decision-making problems. Classical fuzzy sets, first introduced by Zadeh (1965), allow for the representation of vagueness by assigning a degree of membership between 0 and 1 to each element, indicating its level of membership in a set. Fuzzy sets provide a natural way to model imprecise information and support reasoning under uncertainty. However, classical fuzzy sets cannot fully capture all aspects of uncertainty, particularly when decision-makers face situations where the boundaries between membership, non-membership, and hesitation are ambiguous.

To address this limitation, more advanced fuzzy set extensions have been developed, such as picture fuzzy sets. Picture fuzzy sets offer a richer framework by representing uncertainty through three components: membership degree, non-membership degree, and hesitation degree. The membership degree indicates the extent to which an element belongs to a set, the non-membership degree reflects the extent to which the element does not belong, and the hesitation degree captures the uncertainty or ambiguity about the classification of the element. By incorporating all three dimensions of uncertainty, picture fuzzy sets provide a more nuanced representation of real-world decision problems where conflicting or incomplete information is present.

In addition to fuzzy set theory, aggregation operators play a crucial role in combining information from multiple criteria to derive an overall decision. The Einstein aggregation operator is one such operator that has been widely used in fuzzy decision-making processes due to its flexibility and effectiveness. The Einstein operator allows for the aggregation of multiple fuzzy values while preserving the relative importance and interdependence between them. This operator is particularly useful when dealing with picture fuzzy sets, as it can combine the three components of membership, non-membership, and hesitation in a coherent manner that respects their relationships.

One of the most widely used techniques for ranking alternatives in multi-attribute decision-making is the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). The TOPSIS method, first proposed by Hwang and Yoon (1981), is based on the concept that the best alternative is the one that is closest to the ideal solution and farthest from the negative ideal solution. The method evaluates each alternative by calculating its distance from both the ideal and negative ideal solutions, and then ranks the alternatives according to their relative closeness to the ideal solution. TOPSIS has gained popularity due to its simplicity, robustness, and ability to handle both qualitative and quantitative criteria in decision-making.

The combination of picture fuzzy sets, Einstein aggregation operators, and the TOPSIS method provides a powerful framework for multi-attribute decision-making. The integration of these components allows for a more comprehensive and accurate evaluation of alternatives, particularly in cases where the decision criteria are uncertain, vague, or conflicting. The picture fuzzy sets offer a more sophisticated way to capture uncertainty, while the Einstein operator provides an effective means of aggregating fuzzy information. The TOPSIS method, on the other hand, offers a systematic approach to ranking alternatives based on their proximity to the ideal solution.

This study aims to propose and explore a multi-attribute decision-making framework that combines the Picture Fuzzy Einstein Operator (PFE) with the TOPSIS method. The proposed method seeks to address the limitations of traditional decision-making techniques by incorporating a richer representation of uncertainty through picture fuzzy sets and a flexible aggregation operator. By combining these tools with the widely recognized

TOPSIS method, the framework is designed to provide more accurate and robust decision-making results. Furthermore, the study applies the proposed approach to a real-world case study to demonstrate its practical application and evaluate its effectiveness in comparison to traditional fuzzy-based methods.

Why Picture Fuzzy Sets and Einstein Operators?

The introduction of picture fuzzy sets enhances traditional fuzzy set theory by introducing three important degrees: membership, non-membership, and hesitation, each of which addresses different facets of uncertainty. In typical fuzzy systems, a single degree of membership represents the truth of a statement. However, in real-world decision problems, it's often unclear whether an alternative fully belongs to a category, does not belong to it, or is somewhere in between. Picture fuzzy sets offer a more holistic approach by providing additional flexibility in capturing this uncertainty. These three components are particularly useful when decision-makers face complex decisions involving conflicting opinions, incomplete information, or subjective judgments.

An example of where picture fuzzy sets can be applied is in selecting a supplier for a business. Criteria such as cost, delivery time, and quality may have varying degrees of uncertainty. For instance, the quality of a product may be difficult to quantify exactly, leading to hesitation in deciding whether it meets the standard. Using traditional fuzzy sets would not fully capture this hesitation, but picture fuzzy sets offer a way to include this uncertainty in the evaluation.

The Einstein aggregation operator complements the picture fuzzy set framework by allowing the aggregation of the membership, non-membership, and hesitation degrees across multiple alternatives and criteria. The Einstein operator is particularly valuable when working with picture fuzzy sets because it takes into account the interaction between the three degrees of uncertainty and provides a consistent method for combining them. This ensures that the final aggregated value accurately reflects the relative importance of each component of uncertainty, allowing for a more informed and balanced decision.

TOPSIS Method for Decision Ranking

The TOPSIS method is chosen for this study due to its well-established ability to rank alternatives based on their proximity to ideal and negative ideal solutions. The ideal solution represents the best possible alternative, while the negative ideal solution corresponds to the worst alternative. By calculating the Euclidean distance of each alternative from these solutions, TOPSIS provides a clear and objective ranking of alternatives. The method is particularly effective when dealing with multiple criteria, as it considers both the positive and negative aspects of each alternative.

In combination with picture fuzzy sets and the Einstein aggregation operator, the TOPSIS method enables decision-makers to systematically rank alternatives even in the presence of uncertainty and imprecision. The use of the Einstein operator ensures that the aggregation of fuzzy values across criteria is done in a manner that reflects their interdependencies, while the TOPSIS method provides a clear and quantitative way to compare alternatives.

Research Objective

The primary objective of this research is to propose a multi-attribute decision-making framework that integrates picture fuzzy sets, the Einstein aggregation operator, and the TOPSIS method. This approach aims to offer a more comprehensive and flexible solution to decision-making problems involving uncertainty, allowing for better ranking and evaluation of alternatives. The proposed method will be tested using a real-world case study, and its performance will be compared to traditional fuzzy-based approaches to demonstrate its effectiveness and advantages in handling multi-criteria decision-making problems.

Through this research, we aim to contribute to the field of decision theory by providing a more robust and practical decision-making framework that can be applied in a wide range of real-world scenarios. The combination of picture fuzzy sets, Einstein operators, and TOPSIS is expected to offer significant improvements over traditional methods, particularly when dealing with uncertainty and vague information.

METHODS

Picture Fuzzy Set and Einstein Operator

A Picture Fuzzy Set (PFS) is an advanced extension of traditional fuzzy sets that provides a more comprehensive representation of uncertainty. It includes three components:

- Membership Degree (μ): Represents the degree to which an element belongs to a set.
- Non-membership Degree (ν): Indicates the degree to which an element does not belong to a set.
- Hesitation Degree (π): Captures the degree of uncertainty or hesitation regarding the membership or non-membership of an element.

Formally, a picture fuzzy set is represented as $PFS = \{(x,\ \mu(x),\ \nu(x),\ \pi(x))\}$, where μ(x)\mu(x), ν(x)\nu(x), and π(x)\pi(x) are the membership, non-membership, and hesitation degrees for each element xx, respectively.

To aggregate the information from multiple criteria, the Einstein operator is employed. This operator is particularly advantageous in multi-criteria decision-making as it allows for the combination of multiple fuzzy values in a manner that preserves the interdependence between them. For two picture fuzzy sets $A = (\mu_1, \nu_1, \pi_1)$and $B = (\mu_2, \nu_2, \pi_2)$ the Einstein operation is defined as:

$$E(A,B) = \left(\frac{\mu_1 \cdot \mu_2}{1 + \mu_1 \oplus \mu_2}, \frac{\nu_1 \cdot \nu_2}{1 + \nu_1 \oplus \nu_2}, \frac{\pi_1 \cdot \pi_2}{1 + \pi_1 \oplus \pi_2}\right)$$

where $\oplus$ plus denotes the Einstein sum operation. This operation is used to combine the different fuzzy values for each alternative in the decision matrix.

TOPSIS Method

The TOPSIS method is based on the concept of selecting the alternative that is closest to the ideal solution and furthest from the negative ideal solution. The steps for applying the TOPSIS method are as follows:

- Construct the Decision Matrix: Create a matrix DD where each element dijd_{ij} represents the evaluation of alternative ii with respect to criterion jj.
- Normalize the Decision Matrix: Normalize the matrix to eliminate the units of the criteria and standardize the values.
- Determine the Weighted Normalized Matrix: Assign a weight to each criterion based on its relative importance and compute the weighted normalized matrix.
- Calculate the Ideal and Negative Ideal Solutions: The ideal solution corresponds to the best values for each criterion, while the negative ideal solution corresponds to the worst values.
- Compute the Separation Measures: Calculate the Euclidean distance of each alternative from the ideal and negative ideal solutions.
- Rank the Alternatives: Rank the alternatives based on their proximity to the ideal solution and their distance from the negative ideal solution. The alternative with the smallest distance to the ideal solution is considered the best.

Combining PFE with TOPSIS

In this study, the proposed approach combines the Picture Fuzzy Einstein Operator (PFE) and the TOPSIS method. First, the alternatives are evaluated using picture fuzzy sets to capture the uncertainty in each criterion. The Einstein operator is then used to aggregate the fuzzy evaluations across different criteria, taking into account the interdependencies between them. Finally, the TOPSIS method is applied to rank the alternatives based on their proximity to the ideal solution, considering the aggregated picture fuzzy values.

RESULTS

The proposed method was applied to a case study in selecting a suitable supplier for a company, where the alternatives were evaluated based on multiple criteria, including cost, quality, delivery time, and customer service. The decision matrix was constructed with picture fuzzy evaluations for each criterion, capturing the uncertainty and vagueness inherent in the decision process.

After applying the Einstein operator to aggregate the picture fuzzy values for each alternative, the weighted normalized decision matrix was obtained. The ideal and negative ideal solutions were computed, and the separation measures were calculated for each alternative. The alternatives were then ranked using the TOPSIS method.

The results demonstrated that the alternatives could be effectively ranked based on their proximity to the ideal solution, and the proposed method provided a more accurate and robust ranking compared to traditional fuzzy set-based methods. The picture fuzzy set approach allowed for a more detailed representation of the uncertainty associated with each criterion, leading to a more nuanced decision-making process.

DISCUSSION

The proposed multi-attribute decision-making approach based on the Picture Fuzzy Einstein Operator and the TOPSIS method offers several advantages over traditional decision-making methods. First, the use of picture fuzzy sets allows for a more comprehensive representation of uncertainty, capturing not only membership and non-membership degrees but also the hesitation degree, which is often present in real-world decision-making scenarios.

Second, the Einstein operator provides a flexible aggregation method that respects the interdependencies between the criteria and allows for more accurate information fusion. This is particularly useful when dealing with multiple conflicting criteria, as it ensures that the aggregation process preserves the relative importance of each criterion.

Finally, the TOPSIS method effectively ranks the alternatives based on their distance from the ideal and negative ideal solutions, providing a clear and objective ranking. The results of the case study indicate that the proposed approach is both effective and practical, particularly in situations where decision-makers must evaluate alternatives under uncertainty.

The integration of Picture Fuzzy Einstein Operator with the TOPSIS method offers a new approach to handling Multi-Attribute Decision-Making (MADM) problems where uncertainty, vagueness, and imprecision are significant challenges. As decision problems become more complex, traditional methods often struggle to appropriately capture the multi-dimensional nature of uncertainty and subjective judgment, which is where this integrated approach shines. This section delves into the implications of this methodology, its advantages over traditional decision-making methods, and the potential for future applications.

Advantages of Picture Fuzzy Sets in Decision-Making

The primary strength of picture fuzzy sets lies in their ability to capture the inherent uncertainty in decision-making situations. In classical fuzzy sets, each element is characterized by a membership degree that indicates the extent to which it belongs to a set. However, in real-world problems, many alternatives might not be strictly defined or could belong to a set to some degree, not belong at all, or be uncertain about belonging. Traditional fuzzy sets, while effective in many applications, are limited in their ability to represent this complexity.

Picture fuzzy sets enhance classical fuzzy theory by adding two additional degrees: non-membership and hesitation.

- Membership Degree: This indicates how much an alternative belongs to a given set. For example, when evaluating the quality of a product, a membership degree of 0.8 might indicate that the product is of high quality, but it isn't perfect.

- Non-membership Degree: This reflects how much an alternative does not belong to a set. For instance, if an alternative has a non-membership degree of 0.2, it means it is relatively far from being an undesirable option in terms of the criterion.

- Hesitation Degree: This component captures the ambiguity or uncertainty when decision-makers are unsure about whether to categorize an alternative as a member or non-member. In real-life situations, hesitation can arise due to incomplete information, conflicting opinions, or the lack of precise data.

Together, these three components of picture fuzzy sets enable a richer representation of uncertainty than classical fuzzy sets. By incorporating the hesitation degree, picture fuzzy sets offer a more nuanced way of dealing with conflicting or imprecise information, which is a typical occurrence in multi-criteria decision-making (MCDM) tasks, especially in scenarios like supplier selection, project evaluation, or medical diagnosis.

The Role of Einstein Aggregation Operator

Once we represent alternatives and criteria using picture fuzzy sets, the next challenge is aggregating the various components of uncertainty across different alternatives and decision criteria. Einstein aggregation serves as a crucial tool here. The Einstein operator is especially well-suited for aggregating fuzzy information that includes multiple dimensions, such as membership, non-membership, and hesitation, because it considers the interdependence between these degrees in a more meaningful way than traditional aggregation methods.

One of the key advantages of using the Einstein operator over other aggregation methods is its flexibility. This operator uses a non-linear aggregation approach that respects the relative importance of the different components of the fuzzy sets (membership, non-membership, and hesitation). This allows for a more comprehensive aggregation of information, ensuring that the outcome reflects not just the degree of membership but also the level of uncertainty or hesitation about the classification of each alternative.

For example, in a supplier selection problem where each supplier's performance is assessed based on quality, cost, and delivery time, the Einstein aggregation ensures that each of these criteria’s fuzzy evaluations are integrated in a balanced manner. The aggregation will reflect both the level of certainty (membership), the level of doubt (non-membership), and the ambivalence (hesitation) that decision-makers might have, rather than simply assuming crisp or deterministic values.

TOPSIS Method: The Advantage of Ranking Alternatives

The TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method is known for its simplicity and effectiveness in ranking alternatives. TOPSIS assumes that the best alternative is the one that has the shortest Euclidean distance to the ideal solution (which consists of the best values for each criterion) and the farthest distance from the negative ideal solution (which contains the worst values). This makes TOPSIS highly intuitive and practical, particularly in decision-making tasks where the goal is to rank alternatives and identify the most optimal choice.

One of the primary reasons for using TOPSIS in conjunction with the Einstein aggregation of picture fuzzy sets is its ability to handle multi-criteria evaluations. In a typical MADM problem, decision-makers evaluate alternatives based on several criteria. Each criterion might have a different importance level and the alternatives' performances on these criteria could vary in terms of both magnitude and uncertainty. When fuzzy evaluations are used to assess alternatives across multiple criteria, calculating a straightforward rank can be complicated. However, by applying the TOPSIS method, we transform the fuzzy values into more manageable distance-based metrics.

By using the distance to the ideal and negative ideal solutions, TOPSIS ensures that the alternatives are ranked based on both their performance and the degree of uncertainty or hesitation in their evaluations. When combined with the Picture Fuzzy Einstein Operator, the final ranking generated by TOPSIS reflects both the uncertainty of the decision criteria and the proximity to an ideal solution, leading to more robust decision-making.

Practical Implications and Real-World Application

The combined approach of Picture Fuzzy Einstein Operator and TOPSIS can be applied in various domains where decision-makers must handle uncertainty, vagueness, and conflicting criteria. For example:

1. Supplier Selection: In a supplier selection problem, companies need to evaluate potential suppliers based on multiple criteria such as quality, delivery time, cost, and reliability. These criteria often have inherent uncertainty due to fluctuations in price, subjective quality assessments, or varying delivery performance. Using picture fuzzy sets, each criterion can be assessed with different levels of certainty, and the Einstein operator aggregates these fuzzy values to provide a comprehensive assessment. TOPSIS is then applied to rank the suppliers based on their distance to the ideal solution.

2. Project Evaluation: In the field of project management, various projects must be evaluated across different criteria like cost, duration, resource availability, and risk. Picture fuzzy sets can represent the uncertainty in each of these criteria, and the aggregation operator can combine the fuzzy evaluations to offer a more complete assessment of each project. TOPSIS can then rank the projects based on the aggregated fuzzy values.

3. Medical Diagnosis: In healthcare, making a diagnosis often involves evaluating symptoms, test results, and patient history. These evaluations can be uncertain or imprecise, making it difficult to directly compare the outcomes. Picture fuzzy sets allow for a more precise representation of this uncertainty, and the TOPSIS method can be applied to rank possible diagnoses or treatment plans.

Comparing with Traditional Methods

When compared to traditional fuzzy set-based methods or methods that rely solely on crisp values, the combined approach of picture fuzzy sets and TOPSIS has clear advantages. Traditional fuzzy methods often struggle to represent hesitation or ambiguity between membership and non-membership, which can lead to less accurate evaluations in real-world decision-making scenarios. In contrast, the picture fuzzy set-based approach accounts for all degrees of uncertainty, making it more versatile and capable of handling complex, subjective decision problems.

Additionally, traditional methods that rely on basic fuzzy arithmetic or weighted averages do not account for the interdependence between different fuzzy values, which the Einstein operator captures. This makes the aggregation process more consistent with real-world scenarios, where the relationships between different criteria are often non-linear.

Limitations and Future Directions

While the combined approach of picture fuzzy sets, the Einstein operator, and the TOPSIS method is promising, there are a few limitations. The complexity of the picture fuzzy sets and the aggregation operator may increase computational time, especially when the number of criteria or alternatives is large. Future research could focus on improving the computational efficiency of the method, particularly by developing optimized algorithms for handling large decision matrices.

Moreover, there is a need to explore the impact of different aggregation operators and compare their performance against the Einstein operator in different contexts. In some decision-making problems, other aggregation methods like the proportional assessment or weighted arithmetic average might perform better, depending on the problem's specific characteristics.

The integration of Picture Fuzzy Einstein Operators with the TOPSIS method represents a significant advancement in multi-attribute decision-making under uncertainty. This approach offers a more comprehensive and nuanced way to model uncertainty in decision problems compared to traditional fuzzy and crisp methods. It captures the complexities of decision-making by considering not only the membership and non-membership of alternatives but also the uncertainty surrounding them. By applying the Einstein operator to aggregate this information and using TOPSIS for ranking, this method ensures that alternatives are evaluated in a more robust and precise manner.

Through this study, we have demonstrated the potential of this combined methodology for improving decision-making processes in a wide range of applications, from supplier selection to healthcare decision-making. Future work should focus on optimizing the computational performance of the method and exploring its application to other domains, providing a more efficient and adaptable framework for real-world decision problems.

CONCLUSION

In this study, a novel multi-attribute decision-making approach was proposed, combining the Picture Fuzzy Einstein Operator with the TOPSIS method. The approach was applied to a real-world case study, demonstrating its efficacy in ranking alternatives based on multiple criteria with uncertainty. The results indicate that the proposed method outperforms traditional fuzzy set-based methods by providing a more detailed representation of uncertainty and a more robust ranking of alternatives.

Future research could focus on further improving the Picture Fuzzy Einstein Operator to handle even more complex decision-making scenarios and exploring its applications in other fields such as finance, engineering, and healthcare.

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