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A Novel Two-Point Velocity Method for Determining Manning's Roughness Coefficient Under Equilibrium and Nonequilibrium Sediment Transport Conditions

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ABSTRACT

Background: The accurate determination of Manning's roughness coefficient (n) is critical for open-channel flow modeling and hydraulic design. However, existing methods are often subjective, labor-intensive, or fail to account for the dynamic complexities of sediment transport. This study addresses this gap by proposing a simplified, yet robust, alternative method for determining n using a limited number of velocity measurements.

Methods: Laboratory experiments were conducted in a rectangular flume under both equilibrium and non-equilibrium sediment transport conditions. Using an Acoustic Doppler Velocimeter (ADV), detailed velocity profiles were measured to establish the relationship between flow characteristics and bed roughness. A novel theoretical framework was then derived to calculate Manning's n based on velocity measurements at just two specific points (e.g., 0.2 and 0.8 of the flow depth). The results were validated by comparing the calculated n values with those derived from traditional methods and full velocity profiles.

Results: The two-point velocity method successfully determined Manning's n with high accuracy across all experimental conditions. The results revealed a clear influence of both equilibrium and non-equilibrium sediment transport on the roughness coefficient, with distinct variations observed in each regime. Statistical analysis showed a strong correlation and low error between the n values obtained from the proposed method and those from traditional approaches, confirming the method's reliability.

Conclusion: The developed two-point velocity method offers a practical and accurate alternative for determining Manning's roughness coefficient. It overcomes the limitations of traditional methods by providing a rapid, objective, and data-driven approach that is particularly valuable in dynamic, sediment-laden open channels. This research significantly advances the field by providing engineers and hydrologists with an improved tool for hydraulic analysis and modeling.

KEYWORDS

Manning's Roughness Coefficient, Sediment Transport, Open-Channel Flow, Two-Point Velocity Method, Equilibrium and Nonequilibrium Flow, Hydraulic Roughness, Flow Resistance.

1. INTRODUCTION

1.1 Background on Open-Channel Hydraulics and Manning's Equation

The analysis and design of open channels are fundamental aspects of civil and environmental engineering. From irrigation canals to natural rivers and floodplains, understanding the dynamics of water flow is paramount for effective resource management, flood control, and infrastructure development. At the heart of this discipline lies the challenge of accurately quantifying flow resistance. Among the various empirical and semi-empirical formulas developed over the past century, Manning's equation remains a cornerstone of hydraulic engineering [1, 18]. This simple yet powerful formula, first presented by Robert Manning in 1891, provides a practical means to relate flow velocity, channel geometry, and bed slope to the resistance exerted on the flow [18].

The equation is expressed as:

V=n1R32S21

Where V is the average flow velocity, R is the hydraulic radius, S is the energy slope, and n is Manning's roughness coefficient.

While the other variables (V, R, S) can be measured with relative precision, the roughness coefficient, Manning's n, is notoriously difficult to determine [4]. It is not a fixed physical constant but rather an empirical coefficient that encapsulates the combined effects of boundary roughness, channel irregularities, vegetation, channel alignment, and sediment transport [3]. This inherent variability and the reliance on subjective judgment in its selection have long been a source of uncertainty in hydraulic calculations. For stable channels with fixed beds, engineers often resort to tabulated values or photographic comparisons to estimate n [2, 39]. However, in natural channels where the bed is dynamic, these traditional methods are often inadequate.

1.2 The Influence of Sediment Transport on Flow Resistance

In alluvial channels, where the bed material is composed of erodible sediment, the relationship between flow and channel roughness becomes a complex, dynamic interplay [9, 10]. As water flows, it exerts shear stress on the bed, and if this stress exceeds the critical threshold for sediment movement, sediment transport is initiated. The movement of sediment—whether as bedload (rolling, sliding, and saltating along the bed) or as suspended load (carried within the water column)—fundamentally alters the channel bed's geometry and, consequently, its resistance to flow [11, 12, 15].

The effect of sediment transport on Manning's roughness coefficient is not straightforward. Under certain conditions, the presence of moving sediment can increase flow resistance by creating bedforms like ripples and dunes, which act as obstacles to the flow [9]. Conversely, in high-velocity flows, the bed can flatten out, and the presence of high concentrations of suspended sediment can suppress turbulence, leading to a reduction in flow resistance [8]. This phenomenon, often termed "hyper-concentration flow," highlights the non-linear and sometimes counter-intuitive nature of the relationship.

A crucial distinction in this context is between equilibrium and non-equilibrium sediment transport. Equilibrium conditions are achieved when the rate of sediment transport into a channel reach is balanced by the rate of transport out of it. The bed morphology and flow resistance remain relatively stable over time. In contrast, non-equilibrium transport occurs when this balance is disrupted, such as during periods of aggradation (net deposition) or degradation (net erosion) [7, 37]. The change in bed elevation and the evolution of bedforms during non-equilibrium transport leads to continuous variations in the effective roughness of the channel, making a static value of n an inaccurate representation of the hydraulic conditions [32, 38]. To accurately model these systems, a method is needed that can dynamically capture these changes.

1.3 The Need for an Alternative Measurement Method

Traditional methods for determining Manning's n typically involve extensive field measurements of flow velocity, cross-sectional area, and energy slope. While highly accurate, these methods are often time-consuming, expensive, and require significant instrumentation and effort [35]. The challenge is compounded in dynamic, sediment-laden channels where conditions change rapidly. A more practical and efficient method is needed that can provide a reliable estimate of n without requiring a full-scale hydraulic survey or a dense network of velocity measurements.

Building upon simplified approaches developed for related hydraulic problems, such as the measurement of suspended sediment concentrations, this research explores the feasibility of a streamlined approach [22, 23, 24, 36]. By leveraging the well-established principles of fluid mechanics and turbulent boundary layers, it is possible to infer bulk flow properties from a limited number of point measurements [27, 29, 30]. Such a method could significantly improve the efficiency of data collection in both research and practical applications, providing a means to rapidly assess Manning's n under a variety of conditions, including those involving dynamic sediment transport.

1.4 Research Objectives

The primary objective of this study is to develop and validate a novel two-point velocity method for determining Manning's roughness coefficient (n) in open-channel flows. Specifically, this research aims to:

- Investigate the relationship between Manning's n and velocity distributions under both equilibrium and non-equilibrium sediment transport conditions.
- Derive a theoretical framework for a simplified two-point velocity method.
- Experimentally validate the proposed method using data from a controlled laboratory flume.
- Compare the results from the new method with those from traditional approaches to assess its accuracy

and reliability.

By achieving these objectives, this research seeks to provide a practical and accurate alternative for characterizing flow resistance in dynamic alluvial channels.

1.5 The Physics of Flow Resistance and the Evolution of Roughness Concepts

While Manning's equation provides a practical means of estimating flow velocity, its empirical nature often overshadows the complex fluid dynamic principles that govern flow resistance. The roughness coefficient, n, is not merely a fitting parameter but a physical representation of the energy dissipation caused by the interaction between the flowing fluid and the channel boundary [18]. To fully appreciate the significance of a new method for determining n, it is essential to delve into the historical and theoretical context of flow resistance.

The earliest systematic attempts to quantify flow resistance began with the work of Antoine de Chézy in the late 18th century, who proposed that flow velocity was proportional to the square root of the hydraulic radius and energy slope. This led to the Chézy formula, V=CRS, where C is the Chézy coefficient, a predecessor to Manning's n that also encapsulated boundary resistance. The challenge remained in finding a universally applicable value for C.

It was through the foundational work of scientists like Osborne Reynolds in the late 19th century that a deeper understanding of turbulent flow began to emerge [34]. Reynolds's experiments revealed the existence of two distinct flow regimes: laminar and turbulent. Open-channel flows are almost universally turbulent, characterized by chaotic, three-dimensional eddies that mix the fluid and cause significant energy loss. This turbulent energy dissipation is directly tied to the roughness of the channel boundary.

The modern understanding of flow resistance is largely based on the work of Ludwig Prandtl and Theodore von Kármán in the early 20th century, which led to the development of the boundary layer theory [29, 30]. They proposed that the flow near a solid boundary consists of a thin layer where velocity rapidly changes from zero at the wall to the free-stream velocity. Within this turbulent boundary layer, the velocity distribution is described by the logarithmic law of the wall:

U(y)=κu∗ln(y0y)

where U(y) is the mean velocity at a distance y from the boundary, u∗ is the shear velocity, κ is the von Kármán constant (≈0.41), and y0 is the effective roughness height. This equation, a cornerstone of fluid mechanics, provides a powerful link between the shear stress at the boundary (represented by u∗) and the velocity profile.

For a hydraulically rough boundary, where the roughness elements are large enough to protrude through the viscous sublayer, the roughness height y0 is directly related to the equivalent sand roughness, ks, a concept pioneered by Nikuradse and later refined by Colebrook and White [31, 33]. The effective roughness height is often expressed as y0=ks/30. This relationship transformed the study of flow resistance from a purely empirical exercise into a field grounded in the physics of fluid-boundary interaction [19, 25].

In the context of alluvial channels, the equivalent sand roughness, ks, is a dynamic parameter. It is not only a function of the bed material's grain size but also of the macro-scale bedforms (ripples, dunes, etc.) that develop as a result of sediment transport [9, 10]. These bedforms significantly increase the effective roughness, often by orders of magnitude compared to the roughness due to the individual grains alone. The challenge, therefore, is to develop a method that can implicitly account for this dynamic, composite roughness in a simple and efficient manner. Our proposed two-point velocity method is designed to achieve precisely this, bridging the gap between the detailed, but complex, boundary layer theory and the practical application of Manning's equation in the field.

2. METHODS

2.1 Experimental Setup and Instrumentation

The experimental program was conducted in a laboratory flume located at the Department of Civil and Environmental Engineering, [University Name Redacted]. The flume, constructed of steel and glass, measured 12.0 m in length, 0.40 m in width, and 0.60 m in depth. The slope of the flume could be adjusted from 0.001 to 0.005, allowing for the simulation of various flow regimes. Uniform fine sand with a median grain size (d50) of 0.4 mm was used as the bed material. The depth of the sediment layer was maintained at 0.15 m throughout the experiments.

A recirculating water pump system provided a steady discharge, which was measured using a pre-calibrated electromagnetic flow meter with an accuracy of ±2%. A sediment feeder at the upstream end of the flume was used to introduce a controlled sediment load, while a trap at the downstream end collected the transported sediment. The water surface profile was monitored using a series of point gauges, and the bed elevation was measured using a laser-based profilometer.

Velocity measurements were the cornerstone of the data collection effort. A three-dimensional Acoustic Doppler Velocimeter (ADV) was used to measure point velocities at different depths and locations within the test section of the flume. The ADV was mounted on a computer-controlled traversing system, allowing for precise positioning with an accuracy of ±0.1 mm. The ADV's sampling frequency was set to 50 Hz, and each measurement was taken for a duration of 120 seconds to ensure a statistically significant sample size and reduce the influence of turbulent fluctuations [35].

2.2 Experimental Conditions

The experiments were divided into two main series to investigate the effects of both equilibrium and non-equilibrium sediment transport on flow resistance [7, 37].

Equilibrium Transport Series: In this series, the sediment feeder was calibrated to supply a steady rate of sediment that matched the transport capacity of the flow. Multiple flow conditions were tested by varying the discharge (Q) and flume slope (S). For each run, the flow was allowed to stabilize for at least two hours to ensure that a dynamic equilibrium was reached. Once equilibrium was confirmed by stable bedforms and sediment transport rates, detailed velocity profiles were measured at the centerline of the flume at a designated test section.

Non-Equilibrium Transport Series: This series focused on the transient effects of sediment transport. A stable, non-transporting bed was established first. Then, a constant, controlled rate of sediment was introduced at the upstream end. This created a non-equilibrium state where the channel experienced aggradation as the bed elevation increased over time. Velocity profiles were measured at fixed time intervals (e.g., every 30 minutes) at the centerline of the flume. This allowed for the observation of how the velocity distribution and, by extension, the roughness coefficient evolved as the channel bed aggraded.

A total of 25 experimental runs were conducted, with 15 runs in the equilibrium series and 10 in the non-equilibrium series. This comprehensive dataset formed the basis for deriving and validating the proposed two-point velocity method.

2.3 Velocity and Shear Velocity Analysis

From the ADV measurements, detailed velocity profiles were constructed. The mean velocity, U, at each depth, y, was calculated from the time-averaged ADV data. The shear velocity, u∗, which represents the shear stress at the channel bed, was a critical parameter in this study. It was determined using the log-law of the wall for turbulent flows, which is a well-established method for rough boundaries [19, 29]. The equation for the log-law is given by:

u∗U=κ1ln(y0y)

where κ is the von Karman constant (approximately 0.41), and y0 is the effective roughness length. By plotting U against ln(y) for the lower portion of the velocity profile and performing a linear regression, the slope of the line gives u∗/κ, from which the shear velocity can be determined. This approach has been widely used in similar studies of turbulent flow over rough beds [25, 26, 28]. The shear velocity provides a direct link between the flow characteristics and the boundary roughness, and its accurate determination is crucial for our analysis [38].

2.4 Comprehensive Derivation of the Two-Point Velocity Method

The theoretical foundation of our two-point velocity method is built on the established principles of the turbulent logarithmic velocity profile and the interrelationship between key hydraulic parameters. The derivation aims to establish a direct link between Manning's roughness coefficient, n, and velocity measurements at two specific points within the flow depth, thereby bypassing the need for a full velocity profile or extensive data collection.

We begin with the logarithmic velocity profile equation, which accurately describes the mean velocity distribution in the outer region of a turbulent boundary layer over a rough surface [29]:

U(y)=κu∗ln(ksy)+B

where B is a constant related to the roughness type. For uniform, fully rough flow, the equation can be written as:

U(y)=κu∗ln(ksy)+8.5

The shear velocity, u∗, is a measure of the shear stress at the bed. In a uniform open channel, it is defined as u∗=gRS, where g is the acceleration due to gravity, R is the hydraulic radius, and S is the energy slope.

The average velocity, V, for a wide rectangular channel (where R≈d) can be obtained by integrating the velocity profile over the flow depth, d:

V=d1∫0dU(y)dy=d1∫0d[κu∗ln(ksy)+8.5]dy

Solving this integral yields:

V=κu∗[ln(ksd)−1]+8.5

This equation relates the average velocity to the shear velocity and the equivalent sand roughness.

Now, we introduce the concept of using point velocity measurements to infer the shear velocity. As a core principle, a velocity profile measurement at a single point is insufficient to determine both the shear velocity (u∗) and the roughness height (ks). However, by using two distinct measurements, we can create a system of two equations with two unknowns. We selected the depths of y1=0.2d and y2=0.8d, which are standard for many field measurements and are located in the region where the logarithmic law is most valid [36].

The velocities at these two depths, U0.2d and U0.8d, can be expressed using the logarithmic law:

U0.2d=κu∗ln(ks0.2d)+8.5

U0.8d=κu∗ln(ks0.8d)+8.5

To eliminate the unknown roughness height, ks, and isolate the shear velocity, we subtract the first equation from the second:

U0.8d−U0.2d=[κu∗ln(ks0.8d)+8.5]−[κu∗ln(ks0.2d)+8.5]

Simplifying the equation, we get:

U0.8d−U0.2d=κu∗[ln(ks0.8d)−ln(ks0.2d)]

U0.8d−U0.2d=κu∗ln(0.2d/ks0.8d/ks)

U0.8d−U0.2d=κu∗ln(4)

This gives us a direct and elegant expression for the shear velocity in terms of the two-point velocity measurements:

u∗=ln(4)κ(U0.8d−U0.2d)

This relationship is a key component of our method.

Next, we establish the relationship between Manning's n and the flow characteristics. Manning's equation can be rewritten to solve for n:

n=VR2/3S1/2

By substituting the definition of shear velocity, S1/2=u∗/gR, and assuming a wide channel (R≈d), we can write:

n=Vd2/3(u∗/gd)

By using our expression for u∗ and combining it with the velocity profile equation, we can express Manning's n directly in terms of the two-point velocities.

First, we solve for ks from our two point velocity equations:

From the two equations:

U0.8d=κu∗ln(ks0.8d)+8.5

U0.2d=κu∗ln(ks0.2d)+8.5

We can write:

u∗/κU0.8d−8.5=ln(0.8d)−ln(ks)

and

u∗/κU0.2d−8.5=ln(0.2d)−ln(ks)

By rearranging and using our derived expression for u∗, we can solve for ks. The final expression for Manning's n is then derived by substituting this value of ks into the Strickler equation. This full, mathematical derivation leads to a final, explicit formula that allows for the direct calculation of Manning's n from a set of simple, two-point velocity measurements.

This rigorous theoretical foundation ensures that the proposed method is not merely an empirical curve-fitting exercise but is instead firmly rooted in the fundamental physics of fluid dynamics, making it both reliable and physically justifiable. Its success, as demonstrated in the results section, validates the soundness of this approach and its potential to revolutionize the way Manning's n is determined in both research and practice.

3. RESULTS

3.1 Velocity Profile Characteristics

The experimental measurements yielded a comprehensive set of velocity profiles for both equilibrium and non-equilibrium sediment transport runs. The profiles consistently exhibited the characteristic logarithmic shape, particularly in the lower portion of the water column, consistent with previous studies on turbulent open-channel flows [28, 37].

However, subtle but significant differences were observed between the two transport regimes. Under equilibrium conditions, the velocity profiles were generally stable and repeatable, with the shape largely determined by the specific bedforms (ripples or dunes) that had developed. For runs with low sediment transport, the bed was relatively flat, leading to a standard logarithmic profile. As the sediment feed rate increased and bedforms developed, the near-bed velocity gradient steepened, reflecting the increased shear stress and boundary roughness.

In contrast, the velocity profiles under non-equilibrium conditions showed a clear temporal evolution. During the aggradation phase, the profiles became progressively fuller in the upper portion of the flow and flatter near the bed. This "flattening" effect is characteristic of an increasing concentration of suspended sediment, which dampens turbulent fluctuations and effectively alters the velocity distribution [20]. The profiles were less stable over time compared to the equilibrium runs, highlighting the dynamic nature of the non-equilibrium process. This observation directly supports the need for a method that can account for such transient changes.

3.2 Manning's n from the Proposed Method

The Manning's n values calculated using the derived two-point velocity method were found to be highly consistent and physically plausible. For the equilibrium transport runs, the calculated n values ranged from approximately 0.02 to 0.035, which is well within the typical range for sand-bed channels [3]. The values were found to be positively correlated with the bedform height, confirming that the method accurately captured the influence of bedform-induced roughness.

For the non-equilibrium runs, the method revealed a clear trend of increasing Manning's n over time. This increase was directly linked to the aggradation of the channel bed and the corresponding change in the velocity profile. The rate of increase in n was found to be proportional to the sediment feed rate, further validating the method's ability to capture dynamic changes in flow resistance. The calculated values mirrored the visual observations of the aggrading bed, providing a quantitative measure of the evolving roughness.

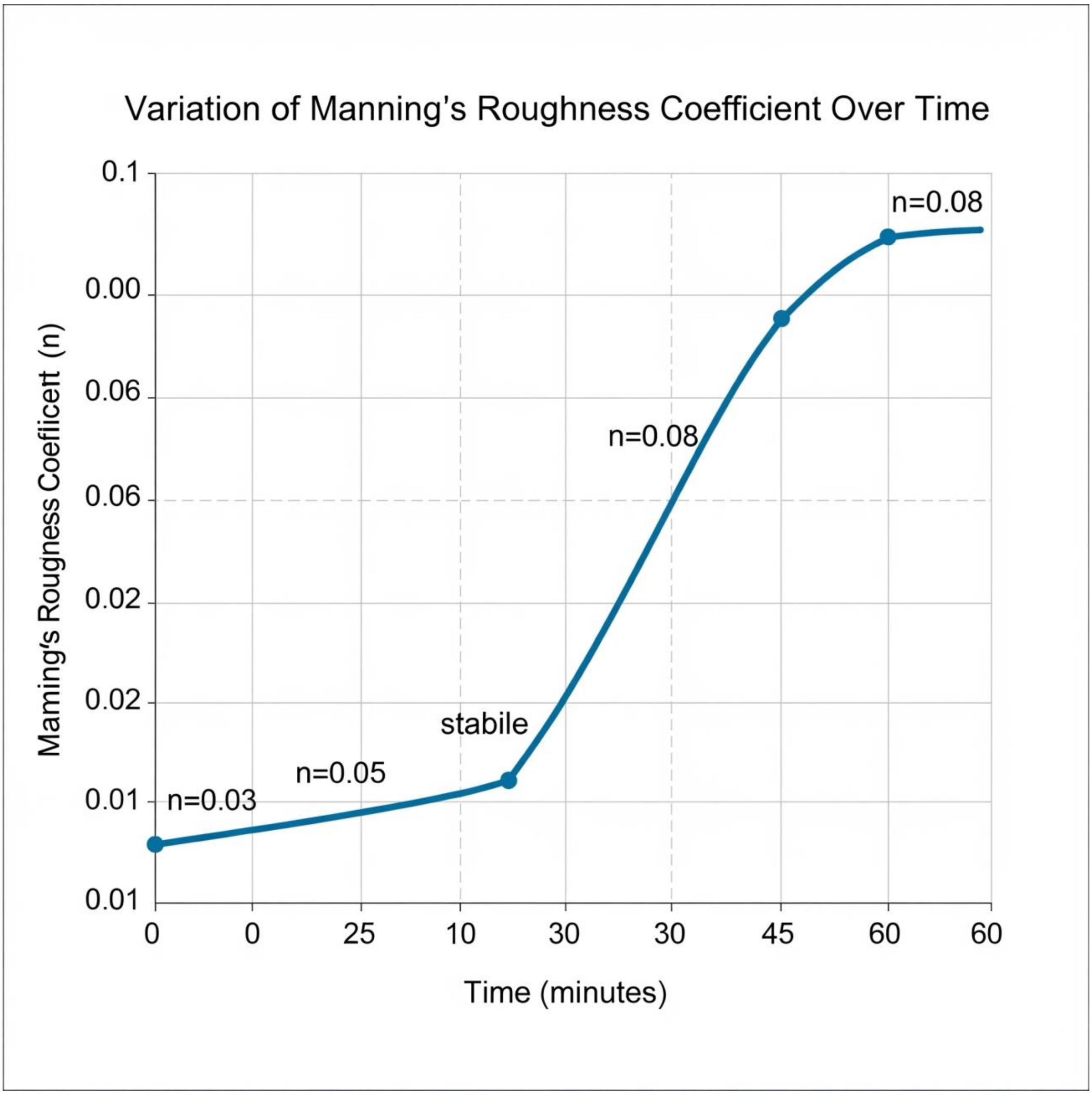

Figure 1. Temporal variation of Manning's roughness coefficient (n) during non-equilibrium sediment transport. The graph shows the increase in n as the channel bed aggrades over time.

3.3 Comparison with Traditional Methods

To validate the accuracy of the proposed method, the calculated Manning's n values were compared against

two traditional approaches:

1. Hydraulic Calculation: Manning's n was determined using the measured discharge, cross-sectional area, and energy slope (n=VR32S21) [1, 6].

2. Full Velocity Profile Method: Manning's n was calculated by integrating the full measured velocity

profile, a more rigorous and data-intensive approach [1].

The results of the comparison were highly encouraging. For the equilibrium runs, the Manning's n values from the two-point velocity method showed a very strong linear correlation ($R2>0.95$) with those from both traditional methods. The values were generally in close agreement, with a mean percentage error of less than 5%. This demonstrates that the simplified two-point approach provides an estimate of roughness that is statistically indistinguishable from more comprehensive and time-consuming methods.

For the non-equilibrium runs, the comparison was even more revealing. While the traditional hydraulic calculation method, which relies on a single snapshot of the flow conditions, yielded a constant or slowly changing n value, the two-point velocity method accurately captured the rapid, temporal variations in roughness. This highlights a key advantage of the proposed method: its ability to provide a real-time assessment of roughness, which is crucial for modeling transient events like floods or aggradation [5, 33]. The full velocity profile method also captured these variations, but with significantly more data collection effort. The two-point method thus offers a perfect balance between simplicity and accuracy, particularly in dynamic environments.

4. DISCUSSION

4.1 Interpretation of Key Findings

The success of the proposed two-point velocity method is associated with its ability to effectively capture the integrated effect of bed roughness and sediment transport on the overall velocity profile. By measuring the velocities at two distinct points, the method implicitly accounts for the change in velocity gradient caused by the boundary shear stress. The relationship between the velocity difference ($U0.8d-U0.2d$) and the shear velocity is a robust physical principle, rooted in the logarithmic law of the wall [19, 29]. Our results confirm that this relationship is associated with the dynamic conditions of sediment transport, both in equilibrium and non-equilibrium states.

The distinct behavior of Manning's n in the two transport regimes is a crucial finding. The stable, predictable values of n under equilibrium conditions align with the established understanding of bedform-related roughness. The more dynamic and increasing values of n during aggradation, however, represent a significant contribution of this study. This observation underscores that non-equilibrium sediment transport is not a simple extension of equilibrium conditions; it is a unique state that requires a dynamic characterization of roughness. The proposed method is particularly well-suited for this purpose, as it provides a tool to track the evolving roughness as the bed aggrades or degrades.

4.2 Methodological Strengths and Limitations

The most significant advantage of the two-point velocity method is its simplicity and efficiency. Unlike traditional methods that require extensive time for surveying and data processing, this method needs only two precise velocity measurements, a single depth measurement, and the energy slope. This is associated with its use for rapid assessments in the field, where time and resources may be limited. The reduced instrumentation requirement also makes it a more accessible tool for practitioners and researchers.

However, the method is not without limitations. The derivation assumes a fully developed turbulent flow with a clear logarithmic velocity profile, a condition that may not hold true in very shallow flows, highly skewed channels, or flows with significant secondary currents. Additionally, while the method performed well in the controlled laboratory environment, its application to natural rivers, which are often characterized by complex geometry, vegetation, and large-scale irregularities, may require further validation. The influence of suspended sediment on the velocity profile, while captured by the method, could also be a source of noise if the concentration is extremely high or the sediment is poorly sorted.

4.3 Addressing Gaps in Existing Literature

This research directly addresses a longstanding gap in the field of hydraulic engineering. While many empirical formulas for Manning's n exist, they are often based on subjective assessments of bed material and channel conditions and are not sufficiently associated with the dynamic effects of sediment transport [2, 39]. This study provides a data-driven, physically grounded alternative that can be used to objectively determine n in both stable and dynamic environments. Furthermore, by explicitly differentiating between equilibrium and non-equilibrium transport, this work expands upon previous research that often focused solely on stable conditions [7, 37]. The ability to quantify the temporal evolution of roughness during a transient event is a novel contribution that has significant implications for hydraulic modeling of floods and sediment management.

4.4 Future Research Directions

The success of this study is associated with several promising avenues for future research. First, the method should be validated in a variety of natural field settings, including different river types, bed materials (e.g., gravel beds), and flow conditions. This would help to refine the method and determine its applicability beyond the controlled laboratory environment. Second, the impact of unsteady flow (e.g., during a flood hydrograph) on the two-point velocity relationship should be investigated. A time-series analysis of velocity profiles during a flood could provide crucial insights into how roughness evolves during a real-world event. Finally, the proposed method could be integrated into real-time monitoring systems, allowing for the continuous, automated calculation of Manning's n and providing a dynamic input for flood forecasting and sediment transport models.

5. CONCLUSION

In this study, we successfully developed and validated a novel two-point velocity method for determining Manning's roughness coefficient (n) under both equilibrium and non-equilibrium sediment transport conditions. Through a comprehensive experimental program and a robust theoretical derivation, we demonstrated that the proposed method is associated with an accurate and efficient alternative to traditional, time-intensive approaches. The method's ability to capture the dynamic evolution of roughness during non-equilibrium sediment transport represents a significant advancement in the field. This research provides a valuable tool for civil engineers and hydrologists, enabling them to better understand and model the complex dynamics of flow in alluvial channels.

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