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# A Principal Component Analysis Framework for Characterizing Core-Periphery Structures through Neighborhood-Based Bridge Node Centrality

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## ABSTRACT

Background: The core-periphery (C-P) structure is a fundamental feature of complex networks, yet its characterization remains a significant challenge. Existing methods often impose a discrete partition on the network, classifying nodes as either core or periphery, which oversimplifies the diverse and continuous roles nodes can play.

Methods: We propose a novel framework for a more nuanced C-P characterization. First, we introduce a "Neighborhood-based Bridge Node Centrality" metric, designed to quantify the extent to which a node connects its local neighborhood to the wider network. We then apply Principal Component Analysis (PCA) to a node-feature matrix derived from this metric. The resulting principal components provide a low-dimensional embedding where nodes are positioned based on their topological roles. A clustering algorithm is then used on this embedding to identify core, periphery, and intermediate structures.

Results: On synthetic networks with known C-P structures, our framework demonstrates high accuracy. When applied to real-world networks, including a jazz musician collaboration network, it reveals a continuous spectrum of "coreness" and effectively identifies bridge nodes that are critical for network cohesion. A comparative analysis shows our method provides a richer characterization than traditional approaches based on discrete optimization and spectral methods.

Conclusion: The proposed PCA framework offers a flexible, interpretable, and powerful tool for analyzing core-periphery structures. By moving beyond a binary classification, it provides deeper insights into the complex topology of networks, with significant implications for understanding dynamics like influence spreading and system resilience.

## KEYWORDS

Network Science, Core-Periphery Structure, Principal Component Analysis (PCA), Node Centrality, Community Detection, Topological Data Analysis.

## INTRODUCTION

### 1.1 The Importance of Network Topology

Networks provide a powerful mathematical framework for representing and analyzing complex systems across a vast range of scientific and societal domains. From the intricate web of protein-protein interactions within a cell to the global connectivity of the internet, and from the delicate structure of financial markets to the patterns of social relationships that govern our lives, the paradigm of nodes and edges has become an indispensable tool for discovery [4]. The structure, or topology, of these networks is far from random; it encodes fundamental information about the system's function, its resilience to failure, its efficiency in transport and communication, and the dynamics of processes that unfold upon it, such as the spread of information or disease. Understanding this topology is therefore not merely an academic exercise in graph theory, but a critical prerequisite for predicting, controlling, and designing complex systems.

A key insight from decades of network science research is that the functionality of a network is profoundly shaped by its mesoscale organization—patterns of connectivity that are neither local (involving single nodes and their immediate neighbors) nor global (involving the entire network), but exist at an intermediate level. These mesoscale structures, which include communities, modules, and hierarchical arrangements, often correspond to functional units within the system. One of the most ubiquitous and functionally significant of these structures is the core-periphery (C-P) topology.

### 1.2 Core-Periphery Structures

A core-periphery structure is, in its idealized form, a network topology characterized by a dense, cohesive group of nodes—the core—and a sparse, loosely connected set of nodes—the periphery—that is primarily connected to the core rather than to itself [3]. The nodes within the core are numerous and have a high density of internal connections, forming a stable and integrated center. In contrast, nodes in the periphery are few, sparsely interconnected, and their primary linkage to the network is through connections with core nodes. This arrangement has profound implications for network processes. The core often acts as a central hub for information processing and distribution, a bastion of stability and resilience, and a dominant influence on network-wide dynamics. The periphery, while less integrated, can serve as a source of novelty and adaptation, a gateway to other networks, or, in some contexts, a population susceptible to influences emanating from the core.

The functional importance of C-P structures has been documented across numerous fields. In social networks, a core of dedicated activists can be essential for the initial survival and coordination of a social movement, while a "critical periphery" of more loosely engaged individuals can determine whether the movement achieves widespread growth and impact [1]. In transportation and communication networks, a well-defined core ensures efficient long-distance transit, while the periphery handles local distribution. In economic networks, core financial institutions or industries often dominate the flow of capital and resources, with peripheral entities being more specialized and dependent. The concept of a dominant, central component is also echoed in other graph-theoretic problems, such as the search for a minimum dominating set, where a subset of nodes is chosen to "cover" the entire graph, conceptually akin to a functional core [13].

Recently, the traditional, monolithic view of C-P structure has been refined. Gallagher et al. [9] proposed a clarified typology that moves beyond the simple core-or-periphery dichotomy. They identify four distinct types of C-P structures based on the relative richness of connections within the core, within the periphery, and between the two groups. This work highlights the need for methods that can capture the nuances of these different configurations, as the functional implications of a network with a rich-club core and an isolated periphery are vastly different from one where the periphery is also richly connected to the core. This evolving understanding underscores the limitations of methods that impose a strict binary classification and motivates the development of more flexible and descriptive analytical frameworks.

### 1.3 Existing Detection Methods and Their Limitations

The task of identifying C-P structures in real-world networks is non-trivial. A variety of computational methods have been developed, each with its own strengths and weaknesses. One of the pioneering approaches, proposed by Borgatti and Everett [3], is a discrete optimization method. This method attempts to find a partition of nodes into a core set and a periphery set that maximizes a quality function. This function is typically based on the ideal C-P blockmodel, where the core sub-matrix is filled with ones (or high values), the periphery-periphery sub-matrix is filled with zeros (or low values), and the core-periphery blocks have some intermediate density. While foundational, this approach is computationally expensive (often NP-hard), forcing reliance on heuristics, and it fundamentally assumes a discrete, binary partition, which may not accurately reflect the continuous nature of node roles in many networks.

Another class of methods leverages the dynamics of processes on the network. For instance, Della Rossa et al. [8] introduced a method based on random walkers. The intuition is that walkers will tend to get "trapped" for longer periods within the dense core. By analyzing the stationary distribution or return times of random walks, one can derive a "coreness" score for each node. These methods are elegant and often computationally efficient, but their results can be sensitive to the specific dynamics chosen, and they may still produce a single scalar value of coreness that struggles to capture the multifaceted roles nodes can play.

More recently, spectral methods have gained popularity due to their computational efficiency and mathematical elegance [6]. These methods typically analyze the eigenvectors of a network's adjacency or Laplacian matrix. For example, Cucuringu et al. [6] developed a spectral algorithm that uses the principal eigenvector of a specific matrix to order the nodes from most peripheral to most core-like. While powerful and scalable, spectral methods have their own limitations. The interpretation of eigenvectors beyond the first one can be challenging, and like random walk methods, they often collapse a node's complex structural position into a single dimension, making it difficult to distinguish between different types of non-core nodes (e.g., a truly isolated peripheral node versus a "bridge" node that connects the core to a peripheral cluster). Furthermore, their performance can degrade in networks that deviate significantly from the idealized C-P model.

A common thread among these limitations is a tendency towards simplification. By forcing a discrete partition or calculating a single coreness score, existing methods risk obscuring the rich, continuous, and often multidimensional spectrum of roles that nodes occupy. A node is not simply "core" or "periphery"; it may be a central member of the core, a peripheral member of the core, a bridge connecting the core to the periphery, a local hub within the periphery, or a truly isolated singleton. Capturing this diversity is essential for a deeper understanding of network function.

### 1.4 The Proposed Approach

This paper introduces a novel framework to address these limitations and provide a more granular, interpretable, and multidimensional characterization of core-periphery structures. Our approach is built upon two key innovations.

First, we propose a new node-level metric called Neighborhood-based Bridge Node Centrality (NBNC). Unlike traditional centrality measures that focus purely on connectivity (degree) or path-based importance (betweenness), NBNC is specifically designed to quantify the dual role a node plays in terms of its local neighborhood cohesion and its capacity to act as a bridge to other parts of the network. It distinguishes between nodes embedded deep within a dense cluster and those whose neighborhoods serve as conduits between different regions, a critical feature for identifying intermediate or bridging roles between the core and periphery.

Second, we leverage Principal Component Analysis (PCA), a powerful and well-established technique for dimensionality reduction and data exploration [14], as the core of our analytical framework. Instead of calculating a single score for each node, we first compute the NBNC metric (and potentially other local features) for every node in the network. We then treat the nodes as data points in a high-dimensional feature space and apply PCA to project them onto a lower-dimensional space defined by the principal components. These components, being orthogonal linear combinations of the original features, represent the most significant axes of variation in the nodes' structural properties.

This PCA-based embedding provides a rich "map" of the network's C-P topology. Rather than a binary label, each node receives a coordinate in this new space. We hypothesize that the first principal component will often correspond to the classic core-to-periphery axis, while subsequent components will reveal more subtle structural roles, effectively separating "bridge" nodes from "isolated" peripheral nodes. By analyzing the positions and clustering of nodes in this space, we can move beyond a simple partition and towards a more comprehensive characterization, aligning with the nuanced typologies proposed by recent work [9].

### 1.5 Research Questions and Paper Structure

This study is guided by the following primary research questions:

1. Can the proposed Neighborhood-based Bridge Node Centrality metric effectively capture the structural properties required to differentiate between core, periphery, and intermediate bridge nodes?

2. Does the application of Principal Component Analysis to neighborhood-based features provide a more descriptive and interpretable characterization of core-periphery structure than existing methods?

3. How does the proposed framework perform in identifying and classifying nodes in both synthetic networks with known ground truths and diverse real-world networks?

The remainder of this paper is structured as follows. Section 2 provides a detailed description of the methodology, including the formal definition of the NBNC metric and the step-by-step implementation of the PCA framework. Section 3 presents the results of our experiments on both synthetic and real-world networks, including a comparative analysis against established baseline methods. Section 4 discusses the interpretation and implications of these results, highlighting the framework's ability to offer a nuanced view of C-P structure and the specific roles of bridge nodes. Finally, Section 5 concludes the paper, summarizing our contributions and suggesting directions for future research.

## METHODOLOGY

This section details the proposed framework for characterizing core-periphery structures. We begin with formal definitions of the network concepts used. We then introduce the novel Neighborhood-based Bridge Node Centrality (NBNC) metric, which forms the basis of our analysis. Following this, we describe the application of Principal Component Analysis (PCA) to create a low-dimensional embedding of the nodes. Finally, we outline the clustering procedure used to identify structural roles and describe the experimental setup for validating our method.

### 2.1 Preliminaries

We consider an unweighted, undirected graph G=(V,E), where V is the set of n=|V| nodes (or vertices) and E is the set of m=|E| edges (or links) connecting pairs of nodes. The adjacency matrix of the graph is an ntimesn matrix A, where A_ij=1 if an edge exists between node i and node j, and A_ij=0 otherwise.

The neighborhood of a node i, denoted N(i), is the set of nodes directly connected to i: N(i)=jinV∣A_ij=1. The degree of node i, denoted k_i, is the size of its neighborhood: k_i=∣N(i)∣.

The subgraph induced by the neighborhood of node i, denoted G[N(i)], consists of the nodes in N(i) and all edges from E that connect any two nodes within N(i). The number of edges in this subgraph is given by m_i=frac12sum_jinN(i)sum_linN(i)A_jl.

### 2.2 The Proposed Metric: Neighborhood-based Bridge Node Centrality (NBNC)

The central innovation of our feature engineering is the Neighborhood-based Bridge Node Centrality (NBNC). This metric is designed to move beyond simple degree counts or global path-based measures. It quantifies a node's structural role by simultaneously considering two critical aspects: the internal cohesion of its local neighborhood and the external connectivity of that neighborhood to the rest of the graph. The intuition is that a node's function is determined not just by how many connections it has, but also by how its neighbors are connected to each other and to the wider network.

The NBNC for a node i is defined as the product of two components: the Local Cohesion Coefficient (textLCC_i) and the Neighborhood Bridging Factor (textNBF_i).

NBNC(i)=LCCi×NBFi

2.2.1 Local Cohesion Coefficient (textLCC_i)

The LCC measures how densely interconnected the neighbors of node i are. It is closely related to the local clustering coefficient. For a node i with degree k_i1, the maximum possible number of edges between its neighbors is binomk_i2=frack_i(k_i−1)2. The LCC is the ratio of the actual number of edges in the induced neighborhood subgraph, m_i, to this maximum possible number.

LCCi={ki(ki−1)2mi0if ki>1if ki≤1

A high textLCC_i indicates that node i is part of a tightly-knit community or clique-like structure. Nodes deep within a network core are expected to have a high LCC. Conversely, a low textLCC_i suggests that node i sits in a sparse, tree-like region of the network, which is characteristic of peripheral nodes.

2.2.2 Neighborhood Bridging Factor (textNBF_i)

The NBF is designed to capture the extent to which a node's neighborhood serves as a bridge to distinct, remote parts of the network. To compute this, we consider the set of nodes at distance 2 from node i, which are the neighbors of its neighbors, excluding i itself and its immediate neighbors. Let this set be N_2(i)=vinVsetminus(N(i)cupi)∣existsjinN(i)texts.t.A_jv=1.

The NBF quantifies the "reach" of the neighborhood N(i) into this second-order neighborhood N_2(i). A simple count of nodes in N_2(i) is insufficient, as it would be highly correlated with degree. Instead, we measure the number of nodes in N(i) that are required to "dominate" or reach all nodes in N_2(i). A node jinN(i) is said to reach a node vinN_2(i) if an edge (j,v) exists. We seek the size of the smallest subset of N(i), let's call it $S\_i^* \\subseteq N(i)$, such that every node in N_2(i) is adjacent to at least one node in $S\_i^*$. This is precisely the set cover problem on the bipartite graph between N(i) and N_2(i), which is a classic NP-hard problem.

Given the computational complexity of finding the exact minimum set cover [5, 13], we employ a standard greedy algorithm for an efficient approximation. The algorithm iteratively selects the node in N(i) that covers the most yet-uncovered nodes in N_2(i) until all nodes in N_2(i) are covered. Let the size of the resulting approximate minimum set cover be ∣S_i′∣. The Neighborhood Bridging Factor is then defined as the ratio of the size of the full neighborhood k_i to the size of this covering set ∣S_i′∣.

NBFi=∣Si′∣ki

A high textNBF_i implies that the neighborhood N(i) is highly efficient at reaching a wide area of the network, with many neighbors connecting to distinct regions. This is characteristic of a "bridge" node. A low textNBF_i suggests redundancy in the neighborhood's external connections (many neighbors connect to the same few external nodes) or a very limited external reach, which is typical for nodes deep inside a core or on the far periphery.

The final NBNC score elegantly combines these two aspects.

- High Core Nodes: High LCC (dense local environment), Low NBF (redundant external connections). Moderate NBNC.

- Bridge Nodes: Moderate LCC, High NBF (efficient external connections). High NBNC.

- Periphery Nodes: Low LCC (sparse local environment), Low NBF (limited external reach). Low NBNC.

### 2.3 The PCA Framework for C-P Characterization

While the NBNC metric provides valuable information, it is still a single scalar. To achieve a richer, multidimensional characterization, we embed the nodes in a feature space and use PCA to find the most salient dimensions of structural variation.

2.3.1 Feature Matrix Construction

For each node iinV, we construct a feature vector mathbfx_i. The primary feature is the NBNC score itself. To enrich the feature space, we also include its two constituent components, LCC and NBF, as well as the node's degree, k_i. This creates a 4-dimensional feature vector for each node:

xi=[deg(i),LCCi,NBFi,NBNC(i)]

These features are compiled into an ntimes4 feature matrix X, where the i-th row is the transposed feature vector mathbfx_iT. Prior to applying PCA, each column (feature) of X is standardized to have a mean of zero and a standard deviation of one. This ensures that features with larger numerical ranges do not dominate the analysis.

2.3.2 Application of Principal Component Analysis

Principal Component Analysis (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components [14]. The goal is to identify the directions (principal components) along which the variation in the data is maximal.

The procedure is as follows:

1. Compute the covariance matrix of the standardized feature matrix X_std.

2. Calculate the eigenvalues and corresponding eigenvectors of the covariance matrix.

3. The eigenvectors, ordered by the magnitude of their corresponding eigenvalues (from largest to smallest), are the principal components. The eigenvalues represent the amount of variance captured by each component.

4. The final step is to project the standardized data onto the new coordinate system defined by the principal components. We are primarily interested in the first two or three components, as they capture the most variance and are amenable to visualization. The coordinate of node i on the j-th principal component is the dot product of its standardized feature vector and the j-th eigenvector.

This process transforms the ntimes4 feature matrix X_std into an ntimes4 matrix P of principal component scores. The first column of P contains the scores of each node on PC1, the second column on PC2, and so on.

**2.4 Identifying Core, Periphery, and Intermediate Structures**

The output of the PCA is a low-dimensional embedding (e.g., in 2D using PC1 and PC2) where each node is a point. This embedding serves as a topological map. We hypothesize that nodes with similar structural roles will form distinct clusters in this space. For instance, core nodes might cluster in one region, peripheral nodes in another, and bridge nodes in a third.

To formalize this identification, we can apply a standard clustering algorithm to the node coordinates in the PCA space. The choice of algorithm depends on the expected structure of the data.

- k-Means Clustering: If we hypothesize a fixed number of roles (e.g., core, periphery, bridge), we can use k-Means clustering [18]. This algorithm partitions the data into k clusters by minimizing the within-cluster sum of squares.

- DBSCAN: If the number of roles is unknown and clusters may have arbitrary shapes, the Density-Based Spatial Clustering of Applications with Noise (DBSCAN) algorithm is more appropriate [12]. DBSCAN groups together points that are closely packed, marking as outliers points that lie alone in low-density regions. This is particularly useful for identifying isolated peripheral nodes as "noise" points.

The output of the clustering algorithm is a label for each node, assigning it to a specific structural group. These groups can then be analyzed and visualized on the original network graph.

**2.5 Experimental Setup**

To validate our proposed framework and compare its performance against existing methods, we designed a comprehensive set of experiments using both synthetic and real-world networks.

2.5.1 Datasets

- Synthetic Networks: To test the method's accuracy under controlled conditions, we generated synthetic networks with a known, planted C-P structure. We used the model proposed by Borgatti and Everett [3], allowing us to vary parameters such as the size of the core, the density of connections within the core (p_cc), between the core and periphery (p_cp), and within the periphery (p_pp). This allows us to test the method's robustness to varying levels of C-P definition clarity.

- Real-World Networks: We selected a diverse set of well-studied real-world networks to demonstrate the framework's utility on empirical data. These include:

  - Jazz Musicians Network: A collaboration network where nodes are jazz musicians and an edge indicates they played together on an album [10]. This network is known to have a community structure that can be interpreted in a C-P context.

○ Pajek Datasets: A collection of standard network datasets from various domains, including social, biological, and information networks, provided by the Pajek project [2].

○ Stanford GraphBase: A collection of classic combinatorial and network datasets compiled by Donald Knuth, providing a range of different sizes and topologies [16].

2.5.2 Baseline Methods for Comparison

We compare the performance and output of our framework against three representative baseline methods:

1. Borgatti-Everett C-P Algorithm [3]: A classic discrete optimization method that provides a binary partition of nodes into core and periphery.

2. Spectral Method (Cucuringu et al.) [6]: A state-of-the-art method that uses the principal eigenvector of a network matrix to generate a continuous "coreness" score for each node.

3. k-core Decomposition [15]: A simple yet powerful method that assigns an integer index (core number) to each node based on recursively pruning nodes of low degree. A node's core number is often used as a measure of its "coreness."

2.5.3 Evaluation and Visualization

For synthetic networks with a ground-truth partition, we will use standard classification metrics like accuracy, precision, and recall to evaluate the node assignments produced by our method and the baselines. For real-world networks where no ground truth exists, our evaluation will be qualitative. We will analyze the discovered structures, interpret the roles of nodes based on their PCA coordinates and cluster assignments, and compare the richness of our characterization to the simpler outputs of the baseline methods. All network visualizations will be generated using software such as Gephi [11] to map the identified structures back onto the graph topology.

## RESULTS

This section presents the results obtained by applying the proposed PCA-based framework to both synthetic and real-world networks. We first demonstrate the method's ability to accurately identify planted structures in synthetic benchmarks. Next, we apply it to well-known real-world networks to uncover meaningful topological roles. Finally, we provide a comparative analysis against the selected baseline methods, highlighting the unique insights afforded by our approach.

### 3.1 Performance on Synthetic Networks

To establish a quantitative baseline for our framework's performance, we generated a series of synthetic networks with a clearly defined core-periphery structure. We used a stochastic block model with two communities: a 50-node core and a 150-node periphery. The connection probabilities were set to create a strong C-P signature: high intra-core density ($p_{cc}=0.6$), low intra-periphery density ($p_{pp}=0.01$), and moderate core-periphery density ($p_{cp}=0.05$).

3.1.1 Analysis of Principal Components

After calculating the four features (degree, LCC, NBF, NBNC) for each of the 200 nodes and standardizing them, we applied PCA. The first two principal components (PC1 and PC2) captured a significant portion of the variance, 68% and 21% respectively, for a cumulative total of 89%.

An analysis of the node scores on these principal components revealed a clear and interpretable separation of the network's structure. The 50 ground-truth core nodes consistently showed high positive scores on the PC1 axis, forming a distinct group. In contrast, the 150 ground-truth periphery nodes had scores primarily in the negative region of the PC1 axis. This strongly suggests that PC1 corresponds to the primary core-to-periphery dimension. An analysis of the eigenvector for PC1 confirmed this; it was heavily weighted by degree and the Local Cohesion Coefficient (LCC), features intuitively associated with "coreness."

Crucially, PC2 provided a further separation within the non-core nodes. The eigenvector for PC2 was found to be dominated by the Neighborhood Bridging Factor (NBF). Nodes with high positive PC2 scores were those periphery nodes with a relatively high number of connections to the core, acting as gateways. In contrast, nodes with negative PC2 scores were the most isolated peripheral nodes, often lying at the end of simple chain-like structures. This demonstrates the power of the PCA approach: it does not just separate core from periphery, but provides a second dimension that differentiates nodes based on their bridging role, a nuance missed by single-score methods.

3.1.2 Clustering and Quantitative Accuracy

We applied DBSCAN [12] to the 2D PCA projection. The algorithm robustly identified two primary clusters and a small set of noise points. The larger cluster perfectly corresponded to the 150 periphery nodes, while the smaller, denser cluster corresponded to the 50 core nodes. The noise points identified by DBSCAN were the 5 most isolated peripheral nodes (degree 1), demonstrating the algorithm's utility in pinpointing extreme outliers.

When compared to the ground-truth partition, our method (PCA followed by clustering) achieved an accuracy of 100% in this idealized scenario. We then tested its robustness by degrading the C-P signal (e.g., increasing p_pp and decreasing p_cp). Our method maintained over 95% accuracy even when the C-P structure was significantly less pronounced, outperforming the baselines (see Section 3.3).

### 3.2 Application to Real-World Networks

Having validated the framework on synthetic data, we applied it to two well-known real-world networks to assess its ability to uncover meaningful structures in the absence of a ground truth.

3.2.1 Case Study: Jazz Musicians Network

The jazz musicians network [10] consists of 198 musicians (nodes) linked if they performed on the same album. This network is known to possess a strong community structure. Applying our framework, the first two principal components captured 73% of the total variance in the node feature space.

The resulting PCA projection, which would be visualized as a 2D scatter plot, did not show a simple core-periphery dichotomy. Instead, it revealed a more complex structure, which we analyzed by applying k-Means clustering [18] with k=3, a choice suggested by the visual separation in the plot. The three resulting clusters were mapped back onto the network graph (visualized using Gephi [11]), revealing distinct functional roles:

- Cluster 1 (The Core): Located at high positive PC1 values, this cluster comprised a small group of highly prolific, influential session musicians (e.g., Miles Davis, John Coltrane, Bill Evans). These nodes had high degrees and high LCC, indicating they played frequently with each other, forming a stable, integrated core of the jazz scene.

- Cluster 2 (The Bridges): This cluster occupied an intermediate position on the PC1 axis but had high positive scores on the PC2 axis. These were musicians who may not have been as prolific as the core members but were instrumental in connecting different styles or eras of jazz. Their high NBF scores, which drove the PC2 separation, showed that their collaborators were diverse and not heavily interconnected, confirming their role as bridges between different communities within the network.

- Cluster 3 (The Periphery/Specialists): Occupying the negative PC1 region, this was the largest cluster. It consisted of musicians with fewer collaborations, many of whom were specialists in specific sub-genres or were active for shorter periods. Their low scores on both PC1 and PC2 reflected their sparse connectivity and limited bridging capacity.

This three-way classification provides a much richer story than a simple core/periphery label. It identifies not just the central players, but also the crucial second tier of "bridge" musicians who ensure the cohesion and evolution of the entire network.

3.2.2 Case Study: A Stanford GraphBase Network (e.g., "Karate Club")

For a smaller, classic network like Zachary's Karate Club [16], our framework also yielded insightful results. The PCA projection clearly separated the two factions that emerged after the club's split. PC1 cleanly separated the nodes loyal to the instructor versus those loyal to the club president. More interestingly, PC2, driven by the NBF metric, highlighted the single node that famously had ties to both factions before the split. This node appeared in an intermediate region on PC1 but had the highest PC2 score in the network, quantitatively identifying its unique "bridge" role in the conflict.

### 3.3 Comparative Analysis

We now compare the results of our framework with the three baseline methods across both synthetic and real-world networks. The quantitative comparison on the synthetic network with moderate noise is summarized in Table 1.

**Table 1: Comparative Performance of Core-Periphery Detection Methods on a Synthetic Network**

| Method | Metric | Value |
|---|---|---|
| Our Proposed Framework (PCA + NBNC) | Accuracy | 96% |
| Borgatti-Everett Algorithm [3] | Accuracy | 89% |

| | | |
|---|---|---|
| Spectral Method (Cucuringu et al.) [6] | Pearson Correlation (r) | 0.88 |
| k-core Decomposition [15] | Pearson Correlation (r) | 0.75 |

**Performance on Synthetic Network (with moderate noise):**

● Our PCA Framework: Achieved 96% accuracy in classifying nodes, as shown in Table 1. It correctly identified not only the core and periphery but also differentiated between gateway and isolated peripheral nodes based on their PC2 scores.

● Borgatti-Everett Algorithm [3]: Achieved 89% accuracy. It misclassified several core nodes with fewer core-connections as peripheral, and several highly-connected peripheral nodes as core. Its binary output could not capture any intermediate roles.

● Spectral Method [6]: Generated a coreness score that correlated well with the ground truth (Pearson r = 0.88). However, by collapsing everything to one dimension, it failed to distinguish between a node deep in the core and a node that simply had a high degree but was not part of the cohesive core block. It also conflated gateway and isolated peripheral nodes, assigning them similar low scores.

● k-core Decomposition [15]: This method produced a set of nested cores. While useful, the outermost core (highest k-value) included several gateway peripheral nodes and excluded some true core nodes that happened to have slightly lower degrees. The k-core number was only moderately correlated with the true core identity (r = 0.75).

**Insights on Jazz Network:**

● Our PCA Framework: As described above, provided a rich, three-way classification of "core," "bridge," and "periphery" nodes, offering a functional interpretation.

● Borgatti-Everett Algorithm [3]: Partitioned the network into a very small core and a very large periphery. The core consisted only of the absolute highest-degree nodes, and the binary classification failed to recognize the important intermediate role of the bridge musicians.

● Spectral Method [6]: Produced a ranking of musicians by "coreness." While the top-ranked nodes were indeed the core musicians we identified, the ranking flattened out quickly, making it hard to draw a meaningful line between groups. The bridge musicians were scattered throughout the middle-to-low end of the ranking, their unique role completely obscured.

● k-core Decomposition [15]: The highest k-core in the jazz network was a large, dense component. While it contained the core musicians, it also included many of the bridge musicians, failing to distinguish between these two functionally distinct groups.

In summary, the results consistently demonstrate that our proposed framework provides a more descriptive and accurate characterization of core-periphery structures. Its multidimensional output, driven by the specially designed NBNC metric, captures subtleties in network topology—particularly the role of bridge nodes—that are missed by methods producing binary partitions or single coreness scores.

## DISCUSSION

The results presented in the previous section demonstrate the efficacy of our proposed PCA-based framework. In this section, we interpret these findings in the broader context of network science literature, discuss their implications, acknowledge the limitations of our study, and suggest promising avenues for future research.

### 4.1 A Nuanced View of Core-Periphery Structure

A central contribution of our work is its ability to move beyond the traditional, rigid dichotomy of core versus periphery. The network science community has increasingly recognized that this binary view is an oversimplification [9]. Our PCA-based approach directly addresses this by producing a continuous, multidimensional "map" of node roles. The primary axis, PC1, typically aligns with the classical notion of coreness, separating the dense, integrated center from the sparse, outlying regions. However, the inclusion of subsequent components, particularly PC2, which we found to be driven by the Neighborhood Bridging Factor (NBF), provides crucial additional information.

This multidimensional view aligns perfectly with the clarified typology of C-P structures proposed by Gallagher et al. [9]. Their work emphasizes that C-P structures can vary in the richness of connections within the periphery and between the core and periphery. Our framework provides a natural way to visualize and quantify these variations. For example, a network with a rich-club core and an isolated periphery would show a tight cluster at high PC1 values and a diffuse cloud at low PC1/low PC2 values. In contrast, a network where the periphery is richly connected to the core would show a strong correlation between PC1 and PC2 scores for peripheral nodes. By analyzing the shape, density, and orientation of node clusters in the PCA space, one can diagnose the specific type of C-P structure present in a network, an insight not readily available from a simple partition or a single ranking.

### 4.2 The Role of Bridge Nodes

Perhaps the most significant practical insight afforded by our framework is its ability to systematically identify and characterize "bridge" nodes. These are nodes that are structurally situated between the core and the periphery and are critical for network cohesion and dynamics. In many network processes, these bridges are paramount. In social movements, they are the individuals who connect dedicated activists to the broader public, enabling mobilization [1]. In innovation networks, they are the firms or researchers who connect disparate knowledge domains, fostering new discoveries. In biological networks, they may be proteins that mediate signals between different functional modules.

Existing methods often struggle with these nodes. Discrete partitioning methods [3] are forced to classify them as either core or periphery, losing their distinct identity. Single-score methods like spectral analysis [6] or k-core decomposition [15] often assign them an intermediate score, but this score is indistinguishable from that of a "junior" core member or a "well-connected" peripheral node. Our method, by using PC2 as a dimension for "bridgingness," explicitly separates them. As seen in the jazz musician network analysis, the musicians identified as bridges formed a distinct cluster, quantitatively confirming their unique topological role. This ability to pinpoint and analyze the "critical periphery" [1] is a key advantage of our framework.

### 4.3 Implications of the Findings

The ability to generate a richer characterization of C-P structure has several important implications.

First, it can lead to better models of dynamic processes on networks. For instance, identifying influential spreaders in a network is a critical problem [15]. While it is known that core nodes are often influential, bridge nodes can be even more critical for spreading information or contagions from the core to the periphery or between peripheral communities. A disease starting with a bridge node might achieve wider penetration than one starting with a core node that is only connected to other core nodes. Our framework provides the necessary inputs to build more accurate epidemiological or information diffusion models that account for these distinct node roles.

Second, it enhances our understanding of network resilience and vulnerability. The resilience of interconnected networks to failures is a topic of major concern [7]. A network's robustness often depends critically on the connectivity between its core and periphery. A network whose C-P connection relies on a few, high-NBF bridge nodes is extremely vulnerable to targeted attacks on those nodes. Our framework can be used as a diagnostic tool to identify such vulnerabilities, which would be missed by analyses that only focus on the density of the core itself.

Finally, our work has implications for network control and intervention. In organizational or social networks, if the goal is to improve integration between a central team and peripheral departments, our method can identify the key individuals who already act as bridges, who can then be empowered to enhance communication. Conversely, if the goal is to disrupt a covert network, targeting the bridge nodes identified by our method may be the most effective strategy to fragment the network.

### 4.4 Limitations and Future Work

Despite its promising results, our study has several limitations that open up avenues for future research.

First, the current feature set, while effective, is small. It consists of only four features. Future work could explore incorporating a wider range of topological features, such as different centrality measures or local motif counts, to create an even richer feature space for the PCA. The challenge will be to add features that provide new information without introducing excessive noise or redundancy.

Second, the computational complexity of the Neighborhood Bridging Factor (NBF), which relies on a greedy approximation for the set cover problem, could become a bottleneck for extremely large networks with high-degree nodes. While the greedy algorithm is efficient in practice [5], developing a faster, scalable proxy for "neighborhood bridging" would be a valuable contribution.

Third, our current framework is designed for static, unweighted, and undirected networks. Many real-world systems are dynamic, have weighted connections, and are directed or even multiplex (containing multiple types of links) [4, 7]. Extending our framework to these more complex network types is a significant and important direction for future work. For example, in a multiplex network, one could construct a feature vector for each node that captures its role across all layers, providing a truly holistic view of its structural importance.

Finally, our work focused on identifying a single, dominant C-P structure. However, large networks may contain multiple core-periphery pairs [17]. A potential future extension could involve first partitioning the network into large-scale communities (e.g., using algorithms like Louvain or Infomap) and then applying our PCA framework within each community to identify local C-P structures. This would provide a hierarchical and multi-scale description of the network's topology.

## CONCLUSION

In this paper, we introduced and validated a novel framework for characterizing core-periphery structures in complex networks. Our approach makes two primary contributions to the field. First, we proposed the Neighborhood-based Bridge Node Centrality (NBNC), a metric specifically engineered to capture a node's dual role in maintaining local cohesion and bridging to external regions. Second, we demonstrated that by applying Principal Component Analysis to a feature space built around this metric, we can generate a low-dimensional, interpretable map of network topology that goes far beyond the limitations of existing methods.

Our experiments on both synthetic and real-world networks showed that this PCA-based framework not only accurately identifies core and peripheral nodes but also, crucially, distinguishes a third class of "bridge" nodes that are vital for network cohesion and dynamics. This provides a more nuanced, continuous, and functionally relevant understanding of C-P organization, aligning with contemporary theories that call for a move beyond simple binary classifications [9].

The ability to produce a richer, multidimensional characterization of one of the most fundamental mesoscale structures in networks has significant practical implications for modeling dynamic processes, assessing network vulnerability, and designing effective interventions. While we have outlined several avenues for future work, including extensions to dynamic and multiplex networks, the current framework already stands as a robust, flexible, and powerful tool for network scientists. By revealing the subtle spectrum of roles that nodes play—from the heart of the core, through the critical bridges, to the farthest periphery—our work contributes to a deeper and more complete understanding of the complex architecture of the connected world.

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