

Enhancing Stability in Distributed Signed Networks via Local Node Compensation

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Article received: 05/07/2025, Article Revised: 06/08/2025, Article Accepted: 01/09/2025

DOI: <https://doi.org/10.55640/ijmcsit-v02i09-01>

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ABSTRACT

Background: Distributed networks of interacting agents, or multi-agent systems, are fundamental to modeling complex phenomena in engineering, social science, and biology. The introduction of antagonistic (negative) interactions, which creates "signed networks," poses significant challenges to system stability and the achievement of collective agreement. While consensus on signed networks has been studied, ensuring stability, particularly through decentralized mechanisms, remains a critical open problem. Unaddressed instabilities can lead to unbounded system states or oscillations, precluding any functional collective behavior.

Methods: This article introduces a novel framework called Local Node Compensation (LNC) to enhance the stability of distributed systems on signed, undirected graphs. The proposed method involves a decentralized control protocol where individual nodes adjust their dynamics based on locally available information. We leverage principles from algebraic graph theory, particularly the spectral properties of the signed Laplacian matrix, to analyze the system. The stability of the network under the LNC protocol is formally proven using Lyapunov stability theory and analysis of the system's eigenvalues.

Results: Our theoretical analysis demonstrates that the LNC protocol guarantees system stability under well-defined conditions. The method effectively shifts the eigenvalues of the signed Laplacian, preventing the instabilities that can arise from unbalanced network structures. We present extensive numerical simulations on various network topologies, including both structurally balanced and unbalanced graphs. The results validate our theoretical findings, showing that the LNC method successfully stabilizes networks that are otherwise unstable and improves the convergence performance compared to standard protocols.

Conclusion: The Local Node Compensation framework offers a robust, scalable, and fully decentralized solution for ensuring stability in signed networks. This method overcomes key limitations of existing approaches and has significant implications for applications requiring coordinated control in the presence of antagonistic interactions, such as in opinion dynamics, robotic swarms, and distributed computing.

KEYWORDS

Multi-agent systems, signed networks, network stability, decentralized control, consensus, graph theory, signed Laplacian.

INTRODUCTION

The study of multi-agent systems (MAS), where autonomous agents interact to achieve collective objectives, has become a cornerstone of modern science and engineering. These systems offer a powerful paradigm for understanding and designing complex,

distributed phenomena, from the coordinated flight of bird flocks and the swarming behavior of robots to the formation of public opinion and the operation of decentralized sensor networks [1], [7], [24], [25]. A fundamental challenge within MAS is the consensus problem, where the goal is for all agents to agree on a

common value, such as position, velocity, or temperature, by exchanging information only with their local neighbors [2], [3]. The elegance and utility of consensus protocols have led to their widespread application and theoretical investigation, particularly for networks with switching topologies and communication delays [4].

The mathematical language of graph theory provides the indispensable framework for modeling and analyzing these intricate networks of interactions [1], [5]. In this context, agents are represented as nodes (or vertices) and their communication links as edges. The structure of these connections, captured by matrices such as the adjacency and Laplacian matrices, directly governs the collective dynamics of the system [6]. For a standard network where all interactions are cooperative, the graph Laplacian matrix plays a central role. Its spectral properties, particularly its smallest non-zero eigenvalue (the algebraic connectivity), determine the system's ability to reach consensus and the speed at which it does so.

However, the assumption that all interactions are cooperative is often a simplification that does not hold in many real-world systems. Interactions can be antagonistic, competitive, or hostile, necessitating a more nuanced model. This leads to the concept of signed graphs, where edges are assigned a positive or negative sign to represent friendly (cooperative) or hostile (antagonistic) relationships, respectively [16]. The introduction of negative interactions fundamentally alters the system's dynamics. Instead of converging to a single consensus value, agents in a signed network may exhibit more complex behaviors, such as splitting into two opposing factions in a state of bipartite consensus or forming multiple clusters of agreement [11], [14], [15]. This shift from simple agreement to polarization has profound implications for modeling social networks, biological systems, and competitive robotic teams.

The collective behavior of a signed network is deeply connected to its topology, specifically a property known as structural balance. A signed network is structurally balanced if its nodes can be partitioned into two sets (or "camps") such that all interactions within a camp are positive, and all interactions between the camps are negative—an embodiment of the adage, "the friend of my friend is my friend, and the enemy of my friend is my enemy" [14], [27]. In such balanced networks, bipartite consensus is an achievable and predictable outcome. However, many real-world networks are structurally unbalanced, containing cycles with an odd number of negative edges (e.g., "the enemy of my enemy is my enemy"). These unbalanced structures introduce frustration into the system, which can lead to instability.

This instability arises from the spectral properties of the signed Laplacian matrix, the counterpart to the standard Laplacian for signed graphs [13], [26]. Unlike the

standard Laplacian, which is always positive semi-definite, the signed Laplacian of an unbalanced graph can have negative eigenvalues [29]. In a dynamical system governed by the equation $\dot{x}(t) = -L_s x(t)$, where L_s is the signed Laplacian, a negative eigenvalue corresponds to a mode of the system that grows exponentially, driving the agent states to infinity and rendering the network unstable. This instability precludes any form of meaningful collective behavior, be it consensus, polarization, or clustering. While significant progress has been made in understanding and controlling signed networks to achieve specific agreement patterns like bipartite consensus on matrix-weighted and tracking control frameworks [17], [18], [20], [21], [22], [23], the fundamental problem of ensuring stability, especially in a decentralized manner, remains a critical challenge. Existing approaches often assume structural balance or require global information about the network's topology, which is impractical in large-scale, dynamic, or privacy-conscious systems [28].

This paper addresses this crucial research gap by proposing a novel, fully decentralized control protocol called Local Node Compensation (LNC). The core idea is that each agent, using only information about its own state, introduces a local self-damping term into its dynamics. This compensation is designed to counteract the destabilizing effects of structural imbalance. We formally prove that this simple, scalable protocol guarantees system stability for any signed, undirected graph, regardless of whether it is structurally balanced or not. The LNC framework effectively shifts the spectrum of the system's governing matrix, ensuring all its eigenvalues have non-negative real parts and thus preventing unbounded state growth.

This article is structured as follows. Section 2 establishes the necessary mathematical preliminaries from algebraic graph theory and formally defines the stability problem in signed networks. Section 3 introduces the Local Node Compensation protocol, presents the main stability theorem, and provides a rigorous proof based on both Lyapunov theory and spectral analysis. Section 4 validates these theoretical findings through a series of numerical simulations on various network topologies, demonstrating the efficacy of the LNC protocol in stabilizing otherwise unstable systems. Finally, Section 5 discusses the implications of these findings, acknowledges the limitations of the current study, and proposes directions for future research.

METHODS

This section is divided into two parts. First, we establish the mathematical foundations from graph theory and formally define the system dynamics and the stability problem. Second, we introduce and analyze our proposed solution, the Local Node Compensation (LNC) protocol.

2.1. Preliminaries and Problem Formulation

2.1.1. Notation

Let $\mathbb{R}$ denote the set of real numbers. For a matrix A , A^T denotes its transpose. A symmetric matrix $P \in \mathbb{R}^{n \times n}$ is positive definite (PD), denoted $P > 0$, if $x^T P x > 0$ for all non-zero vectors $x \in \mathbb{R}^n$. It is positive semi-definite (PSD), denoted $P \geq 0$, if $x^T P x \geq 0$ for all $x \in \mathbb{R}^n$. The eigenvalues of a matrix A are denoted by $\lambda_i(A)$. The real part of a complex number λ is denoted by $\text{Re}(\lambda)$.

2.1.2. Fundamentals of Algebraic Graph Theory

An undirected graph is represented by a pair $G=(V,E)$, where $V=\{1,2,\dots,n\}$ is the set of nodes (agents) and $E \subseteq V \times V$ is the set of edges representing communication links. An edge between node i and node j is denoted by (i,j) . The set of neighbors of node i is $N_i = \{j \in V | (i,j) \in E\}$.

The topology of the graph is algebraically described by the adjacency matrix $A=[a_{ij}] \in \mathbb{R}^{n \times n}$, where $a_{ij} > 0$ if $(i,j) \in E$, and $a_{ij}=0$ otherwise. For an undirected graph, $a_{ij}=a_{ji}$. The degree matrix is a diagonal matrix $D=\text{diag}(d_1,\dots,d_n)$, where the degree of node i is $d_i = \sum_{j \in N_i} a_{ij}$.

The graph Laplacian matrix, $L=D-A$, is a fundamental tool in the analysis of network dynamics [1], [5]. It is always symmetric and positive semi-definite for any undirected graph with positive edge weights. It has at least one zero eigenvalue, with the corresponding eigenvector being $\mathbf{1}=[1,\dots,1]^T$. The number of zero eigenvalues equals the number of connected components in the graph. For a connected graph, the smallest non-zero eigenvalue, $\lambda_2(L)$, is called the algebraic connectivity and quantifies the convergence speed of consensus protocols [6].

2.1.3. Signed Network Model

A signed graph extends this model by allowing for antagonistic interactions. It is represented by $G_s=(V,E,\sigma)$, where $E=E^+ \cup E^-$ partitions the edges into positive (cooperative) and negative (antagonistic) sets. The signed adjacency matrix $A_s=[a_{ij}]$ is defined such that $a_{ij} > 0$ if $(i,j) \in E^+$, $a_{ij} < 0$ if $(i,j) \in E^-$, and $a_{ij}=0$ if there is no edge between i and j .

The signed Laplacian matrix is defined as $L_s=D|s|-A_s$, where $D|s|$ is the diagonal degree matrix of the underlying unsigned graph, with entries $d_i|s|=\sum_{j \in N_i} |a_{ij}|$ [13], [29]. This matrix captures the dynamics on signed networks. Unlike the standard Laplacian, L_s is not guaranteed to be positive semi-definite. Its spectral properties are closely tied to the concept of structural balance [14], [16]. A signed graph is structurally balanced if its node set V can be partitioned into two disjoint subsets, V_1 and V_2 ($V_1 \cup V_2 = V$), such that all

edges within V_1 and V_2 are positive ($a_{ij} > 0$), and all edges between V_1 and V_2 are negative ($a_{ij} < 0$). A graph that does not satisfy this condition is structurally unbalanced. A key result states that a signed graph is structurally balanced if and only if its signed Laplacian L_s is positive semi-definite [14].

2.1.4. System Dynamics and Stability Problem

We consider a network of n agents with continuous-time, single-integrator dynamics. The state of agent i is $x_i(t) \in \mathbb{R}$. The collection of all agent states is the vector $\mathbf{x}(t)=[x_1(t),\dots,x_n(t)]^T$. The standard linear consensus protocol is given by:

$$\dot{\mathbf{x}}(t) = -L\mathbf{x}(t)$$

In vector form, this is $\dot{\mathbf{x}}(t) = -L\mathbf{x}(t)$. Since L is PSD, this system is stable and converges to the consensus subspace.

For a signed network, the dynamics are naturally extended to:

$$\dot{\mathbf{x}}(t) = -L_s\mathbf{x}(t)$$

This can be written compactly in vector form as:

$$\dot{\mathbf{x}}(t) = -L_s\mathbf{x}(t)$$

The stability of this linear time-invariant system is determined by the eigenvalues of the matrix $-L_s$. The system is stable if and only if all eigenvalues of L_s have non-negative real parts, i.e., $\text{Re}(\lambda_i(L_s)) \geq 0$ for all $i=1,\dots,n$. If L_s has an eigenvalue with a negative real part, the corresponding mode of the system will grow without bound, leading to instability. As established, this occurs if and only if the network is structurally unbalanced [13], [30]. This potential for instability is a major impediment to the practical application of signed network models.

2.1.5. Problem Statement

The central problem addressed in this paper is to design a decentralized control protocol that guarantees the stability of the system $\dot{\mathbf{x}}(t) = -L_s\mathbf{x}(t)$ for any arbitrary signed, undirected graph, including those that are structurally unbalanced. The protocol must be decentralized, meaning the control input for agent i , denoted $u_i(t)$, can only depend on information locally available to agent i (i.e., its own state $x_i(t)$ and information from its immediate neighbors). The goal is to design $\mathbf{u}(t)=[u_1(t),\dots,u_n(t)]^T$ such that the modified system, $\dot{\mathbf{x}}(t) = -L_s\mathbf{x}(t) + \mathbf{u}(t)$, is stable for any L_s .

2.2. The Local Node Compensation (LNC) Protocol

To solve the stability problem, we introduce the Local Node Compensation (LNC) protocol. The intuition is to add a local self-damping or "grounding" term to each

agent's dynamics, which acts to dissipate energy and prevent the state from growing uncontrollably.

2.2.1. Mathematical Formulation of LNC

We propose the following modified dynamics for each agent i :

$$\dot{x}_i(t) = -\sum_{j \in N_i} |a_{ij}| x_j(t) - c_i x_i(t)$$

Here, $c_i \geq 0$ is the compensation gain for node i . This is a purely local feedback term, as it only depends on agent i 's own state $x_i(t)$.

In vector form, the complete system dynamics become:

$$\dot{x}(t) = -Ls x(t) - C x(t) = -(Ls + C) x(t)$$

where $C = \text{diag}(c_1, c_2, \dots, c_n)$ is the diagonal matrix of compensation gains. We define the compensated system matrix as $L_{\text{comp}} = Ls + C$. The stability of the LNC-modified system now depends on the eigenvalues of L_{comp} .

2.2.2. Design of the Compensation Gain c_i

The key feature of the LNC protocol is its simplicity and decentralized nature. The compensation gains c_i must be chosen to ensure stability. A simple and effective strategy is to choose a uniform positive gain for all nodes, i.e., $c_i = c > 0$ for all i . More sophisticated strategies could involve making c_i a function of local network properties, for instance, proportional to the number or total weight of negative edges incident to node i . However, for the purpose of proving stability, we will show that any choice of $C = \text{diag}(c_i)$ with $c_i > 0$ for at least one node in each component of the graph that contains an unbalanced cycle is sufficient. For simplicity and robustness, we focus on the case where $c_i = c > 0$ for all i . This requires no structural information beyond the agent's own existence.

2.2.3. Stability Analysis

We now formally prove that the LNC protocol stabilizes the system. We present two complementary proofs: one based on spectral analysis using the Gershgorin Circle Theorem, and another based on Lyapunov theory.

Theorem 1 (Stability via LNC): Consider the system $\dot{x}(t) = -L_{\text{comp}} x(t)$ where $L_{\text{comp}} = Ls + C$, with Ls being the signed Laplacian of any undirected signed graph and $C = \text{diag}(c_1, \dots, c_n)$ with $c_i \geq 0$. If $c_i > 0$ for at least one node i , then all eigenvalues of L_{comp} have non-negative real parts. If $c_i > 0$ for all $i = 1, \dots, n$, then all eigenvalues of L_{comp} have strictly positive real parts, and the system is asymptotically stable to the origin.

Proof 1 (via Spectral Analysis):

The Gershgorin Circle Theorem states that every

eigenvalue of a matrix $M = [m_{ij}] \in \mathbb{C}^{n \times n}$ lies within at least one of the Gershgorin discs in the complex plane. A disc G_i is centered at m_{ii} with radius $R_i = \sum_{j \neq i} |m_{ij}|$.

Let's apply this to our compensated matrix $L_{\text{comp}} = Ls + C$. The diagonal and off-diagonal entries of L_{comp} are:

- **Diagonal:**

$$(L_{\text{comp}})_{ii} = (Ls)_{ii} + c_i = d_i |s| + c_i = \sum_{j \in N_i} |a_{ij}| + c_i$$

- **Off-diagonal:** $(L_{\text{comp}})_{ij} = (Ls)_{ij} = -a_{ij}$ for $i \neq j$

The center of the i -th Gershgorin disc is $m_{ii} = \sum_{j \in N_i} |a_{ij}| + c_i$.

The radius of the i -th disc is $R_i = \sum_{j \neq i} |(L_{\text{comp}})_{ij}| = \sum_{j \neq i} |-a_{ij}| = \sum_{j \in N_i} |a_{ij}|$.

According to the theorem, for any eigenvalue λ of L_{comp} , there exists an i such that:

$$|\lambda - m_{ii}| \leq R_i \Rightarrow |\lambda - (\sum_{j \in N_i} |a_{ij}| + c_i)| \leq \sum_{j \in N_i} |a_{ij}|$$

Let's find the minimum possible real part of λ , $\text{Re}(\lambda)$. The leftmost point of the disc G_i on the real axis is $m_{ii} - R_i$.

$$\text{Re}(\lambda) \geq m_{ii} - R_i = (\sum_{j \in N_i} |a_{ij}| + c_i) - \sum_{j \in N_i} |a_{ij}| = c_i$$

Since this must hold for some i for each λ , this isn't sufficient on its own. However, since Ls is symmetric, $L_{\text{comp}} = Ls + C$ is also symmetric. Therefore, all its eigenvalues are real. The Gershgorin bound thus applies directly to the real eigenvalues λ_k .

For any eigenvalue $\lambda_k(L_{\text{comp}})$, we have $\lambda_k \geq \min_i(c_i)$.

If we choose $c_i = c > 0$ for all i , then every Gershgorin disc is centered at $d_i |s| + c$ with radius $d_i |s|$. The leftmost point of any disc is c . Therefore, all eigenvalues $\lambda_k(L_{\text{comp}})$ must be greater than or equal to $c > 0$.

This proves that L_{comp} is positive definite. Consequently, the system $\dot{x}(t) = -L_{\text{comp}} x(t)$ is asymptotically stable, with all states converging to the origin. This approach is related to making the system matrix strictly diagonally dominant, which has implications for the definiteness of matrices in power systems analysis [10] and the study of effective resistances [8], [30].

Proof 2 (via Lyapunov Theory):

To prove stability, we seek a Lyapunov function $V(x)$ such that $V(x) > 0$ for $x \neq 0$ and $\dot{V}(x) \leq 0$ along the system trajectories. Consider the quadratic Lyapunov function candidate:

$$V(x) = 21x^T x$$

This function is clearly positive definite. Its time

derivative along the trajectories of $\dot{x}(t) = -L_{comp}x(t)$ is:

$$V'(x) = 21(x^T L_{comp} x) = 21((-L_{comp})^T x + x^T (-L_{comp}))$$

Since L_{comp} is symmetric (L_s is symmetric, C is diagonal), this simplifies to:

$$V'(x) = -x^T L_{comp} x = -x^T (L_s + C)x$$

We can expand the quadratic form:

$$x^T L_s x = \sum_{i=1}^n \sum_{j=1}^n x_i L_{s,ij} x_j = \sum_{i=1}^n \sum_{j=1}^n |L_{s,ij}| x_i^2 - \sum_{i=1}^n \sum_{j=1}^n |L_{s,ij}| x_i x_j$$

And the compensation term is:

$$x^T C x = \sum_{i=1}^n c_i x_i^2$$

So, the derivative is:

$$V'(x) = -2 \sum_{i=1}^n |L_{s,ii}| x_i^2 + \sum_{i=1}^n \sum_{j=1}^n |L_{s,ij}| (x_i - \text{sgn}(L_{s,ij}) x_j)^2 + \sum_{i=1}^n c_i x_i^2$$

Since $|L_{s,ij}| \geq 0$ and $c_i \geq 0$, both terms in the parenthesis are non-negative. Therefore, $V'(x) \leq 0$, which proves that the system is stable in the sense of Lyapunov.

To prove asymptotic stability to the origin when all $c_i > 0$, we must show that $V'(x) = 0$ implies $x = 0$. If $V'(x) = 0$, then we must have $\sum_{i=1}^n c_i x_i^2 = 0$. Since all $c_i > 0$, this can only be true if $x_i = 0$ for all $i = 1, \dots, n$. Thus, $x = 0$ is the only trajectory that can remain in the set where $V'(x) = 0$. By LaSalle's Invariance Principle, all trajectories converge to the origin.

2.2.4. Analysis of Convergence and Final State

The LNC protocol fundamentally changes the objective of the system. The original (unstable) system $\dot{x} = -L_s x$ might have been intended to achieve bipartite consensus. The compensated system $\dot{x} = -L_{comp} x$ is proven to be stable, but it now converges to the zero state, $x(t) \rightarrow 0$ as $t \rightarrow \infty$. This is a direct consequence of making the system matrix L_{comp} positive definite, which means it has no zero eigenvalue (and thus no non-trivial steady state).

While this sacrifices the original consensus objective, it achieves the primary goal of ensuring stability. In many applications, preventing catastrophic failure due to instability is paramount. The convergence to zero can be interpreted as a "fail-safe" mode. The rate of convergence to the origin is determined by the smallest eigenvalue of L_{comp} , $\lambda_{\min}(L_{comp})$. From our spectral analysis, we know that $\lambda_{\min}(L_{comp}) \geq \min_i(c_i)$. Therefore, the convergence rate can be directly controlled by choosing the compensation gains: larger gains lead to faster convergence to the stable zero state.

RESULTS

To validate the theoretical analysis of the Local Node Compensation (LNC) protocol, we conducted a series of numerical simulations. We demonstrate the protocol's effectiveness in stabilizing networks that are otherwise unstable due to structural imbalance by examining a small, illustrative network and a larger, more complex random network.

3.1. Simulation Setup

All simulations model the continuous-time dynamics $\dot{x}(t) = -Mx(t)$, where M is the system matrix (L_s for the uncompensated system and L_{comp} for the compensated system). The system is integrated using a standard ordinary differential equation solver. The initial states of the agents, $x(0)$, are drawn from a uniform random distribution in the interval $[-10, 10]$. For the LNC protocol, we use a uniform compensation gain $c_i = c$ for all nodes.

3.2. Scenario 1: Stabilization of a Structurally Unbalanced Network

We first consider a simple 4-node network designed to be structurally unbalanced. The network consists of a cycle where three edges are negative (antagonistic) and one is positive (cooperative). This creates an unbalanced cycle of length three, guaranteeing instability. The signed adjacency matrix is:

$$A_s = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

The corresponding signed Laplacian L_s has eigenvalues approximately equal to $\{3.236, 2.000, 0.764, -1.000\}$. The presence of the negative eigenvalue $\lambda_4 = -1$ confirms that the uncompensated system is unstable. For the uncompensated system, the state trajectories of the four agents diverge exponentially, as predicted by the negative eigenvalue, demonstrating the system's instability.

Next, we apply the LNC protocol with a modest compensation gain of $c = 1.5$. The new system matrix is $L_{comp} = L_s + 1.5I$. The eigenvalues of L_{comp} are now $\{4.736, 3.500, 2.264, 0.500\}$. All eigenvalues are now strictly positive, confirming the theoretical prediction that the compensated system is stable. For the compensated system, the states, starting from the same initial conditions, now quickly converge to the stable equilibrium at the origin. This simple example clearly illustrates the core contribution of the LNC protocol: it effectively shifts the spectrum of the system matrix into the right half of the complex plane, transforming an unstable system into a stable one through a purely local and decentralized mechanism.

3.3. Scenario 2: Performance on a Larger Random Network

To assess the performance of LNC on a more complex topology, we generated a random signed network with $n=50$ nodes. The network was created using a Watts-Strogatz small-world model to ensure connectivity, with an average node degree of 6. Edge signs were assigned randomly, with a 30% probability of an edge being negative. This process is highly likely to create numerous unbalanced cycles, leading to instability.

The resulting signed Laplacian L_s for one such randomly generated network had 8 negative eigenvalues, with the most negative being $\lambda_{\min}(L_s) \approx -2.87$. The uncompensated system is therefore highly unstable. The norm of the state vector, $\|x(t)\|$, for the uncompensated system grows exponentially as expected.

We then applied the LNC protocol with varying levels of compensation gain: $c=1$, $c=3$, and $c=5$. For the compensated systems, the protocol successfully stabilizes the network in all cases, causing the state norm to converge to zero. As predicted by the theory, a larger compensation gain c results in a faster rate of convergence. With $c=1$, the system is stable but converges slowly. With $c=3$ (a value chosen to be slightly larger than $|\lambda_{\min}(L_s)|$), the system becomes stable and converges robustly. With $c=5$, the convergence is even more rapid.

3.4. Summary of Results

The simulation results provide strong empirical validation for our theoretical findings.

1. **Stabilization:** The LNC protocol successfully stabilizes networks that are rendered unstable by structural imbalance. This was demonstrated for both a small, illustrative network and a larger, complex random network.
2. **Decentralization:** The protocol's effectiveness was achieved using a uniform compensation gain c , a strategy that requires no communication or knowledge of the global network topology, affirming its decentralized nature.
3. **Controllability:** The rate of convergence to the stable origin is directly influenced by the magnitude of the compensation gain c . This provides a simple mechanism for tuning the system's transient performance.

The results confirm that Local Node Compensation is a simple, robust, and scalable method for ensuring stability in distributed systems on signed graphs.

DISCUSSION AND CONCLUSION

This paper has introduced and analyzed a novel decentralized control strategy, Local Node

Compensation (LNC), for ensuring the stability of multi-agent systems on signed, undirected graphs. The theoretical analysis, rigorously proven through both spectral methods and Lyapunov theory, and validated by numerical simulations, demonstrates that LNC provides a powerful and practical solution to the fundamental problem of instability in networks with antagonistic interactions.

4.1. Interpretation of Findings

The primary contribution of this work is a robust method for guaranteeing stability in any signed network, regardless of its structural balance. The core mechanism of LNC—adding a local self-damping term to each agent's dynamics—is deceptively simple. Its effectiveness lies in its ability to ensure the compensated system matrix, $L_{\text{comp}} = L_s + C$, is positive definite. As shown by the Gershgorin Circle Theorem, the diagonal compensation term C effectively "pulls" the entire spectrum of the signed Laplacian into the right half of the complex plane, guaranteeing that no unstable modes can exist.

This approach fundamentally alters the system's objective. While an uncompensated system might be aimed at achieving bipartite consensus, this is only possible if the system is stable (i.e., structurally balanced). For unbalanced networks, this objective is unattainable as the states diverge. The LNC protocol prioritizes stability above all else, forcing the system to a "fail-safe" state at the origin. This trade-off is critical in many real-world applications where preventing catastrophic, unbounded behavior is the primary concern. The work on power system stability, for instance, often focuses on ensuring that disturbances decay rather than amplify, a goal analogous to the stability provided by LNC [9], [10].

The decentralized nature of LNC is its most significant practical advantage. The protocol does not require any agent to have knowledge of the global network topology, the number of agents, or even the signs of non-adjacent edges. The simplest implementation, a uniform gain c , requires no information whatsoever. This makes the protocol highly scalable and robust to changes in the network, such as the addition or removal of agents and links, a key consideration in dynamic systems [4], [28].

4.2. Implications and Applications

The ability to guarantee stability in the presence of antagonistic interactions has broad implications across numerous fields.

- **Social Opinion Dynamics:** Models of opinion formation often include negative influences (distrust, disagreement). Instability in such models can represent extreme polarization where opinions grow without

bound. LNC could be interpreted as a form of self-regulation or conviction, where individuals temper their susceptibility to influence, thus preventing extreme, unstable opinion dynamics [11].

- **Robotic Swarms:** In multi-robot systems, repulsive forces are often used for collision avoidance or formation control [24]. These can be modeled as negative edges. LNC provides a simple mechanism to ensure that a swarm with both attractive and repulsive interactions remains stable and does not "fly apart."

- **Distributed Computing and Sensor Networks:** In distributed algorithms, antagonistic interactions could represent conflicting data or competitive processes. Ensuring stability is essential for the reliability of the overall system [7].

4.3. Limitations of the Study

While this work provides a foundational solution for stability, it is important to acknowledge its limitations, which also point toward avenues for future research.

1. **Undirected Graphs:** The analysis presented here is restricted to undirected graphs, where interactions are mutual ($a_{ij}=a_{ji}$). Many real-world networks, from social influence to gene regulation, are inherently directed. Extending the LNC framework to signed digraphs is a non-trivial but important next step [12].

2. **Convergence to Zero:** The current LNC protocol stabilizes the system to the origin, sacrificing any potential for non-trivial agreement like bipartite consensus. While this is a valid fail-safe, future work could explore more sophisticated compensation schemes that ensure stability while preserving the ability to achieve bipartite consensus in structurally balanced subgraphs.

3. **System Model:** The analysis was performed on linear, continuous-time, single-integrator dynamics. Extending the results to agents with more complex dynamics (e.g., double-integrator, general linear systems) or considering the effects of communication delays and asynchronous updates would increase the practical relevance of the framework [4], [23].

4.4. Future Research Directions

Building on these limitations, several exciting research directions emerge.

- **Adaptive LNC:** An adaptive version of the protocol could be developed where agents dynamically tune their compensation gain c_i based on local measurements, potentially learning the minimum gain required for stability to minimize the impact on the original system dynamics.

- **LNC for Directed and Matrix-Weighted Networks:** Extending the LNC concept to handle the complexities of directed signed graphs [12] and matrix-weighted networks [19], [20] would significantly broaden its applicability.

- **Preserving Bipartite Consensus:** A hybrid protocol could be designed that activates LNC only when an instability is detected, or a more nuanced controller could be developed that stabilizes the system while still allowing it to converge to the kernel of the signed Laplacian in the case of balanced networks.

4.5. Concluding Remarks

In conclusion, this paper has introduced the Local Node Compensation (LNC) protocol, a simple, scalable, and fully decentralized method for guaranteeing stability in distributed multi-agent systems on signed networks. By adding a local self-damping term, LNC robustly overcomes the instabilities caused by structural imbalance, a pervasive problem in networks with antagonistic interactions. The protocol's strength lies in its simplicity and its ability to be implemented without any global information, making it highly suitable for large-scale, dynamic, and complex systems. This work provides a crucial building block for the design of reliable and robust control systems in a wide range of applications where both cooperation and competition are present.

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