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LOCAL NODE COMPENSATION FOR ENHANCED STABILITY IN DISTRIBUTED SIGNED NETWORKS

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ABSTRACT

Multi-agent systems exhibiting both cooperative and antagonistic interactions, often modeled as signed networks, present unique challenges for achieving system stability and desired collective behaviors. Traditional consensus algorithms, primarily designed for purely cooperative networks, often fail in the presence of negative links, leading to phenomena like polarization or divergence. This article introduces a novel distributed stabilization strategy for signed networks based on local node compensation, effectively adding self-loops to individual agents. By leveraging this compensatory mechanism at each node, agents can autonomously adjust their dynamics to counteract the destabilizing effects of antagonistic connections, thereby promoting system stability. We detail the theoretical framework for incorporating self-loop compensation into standard agent dynamics and analyze its impact on the network's spectral properties. Hypothetical results demonstrate that this localized intervention significantly enhances the stability margin and convergence characteristics, offering a scalable and implementable solution for maintaining coherent behavior in complex signed multi-agent environments.

Keywords: Distributed signed networks, local node compensation, network stability, consensus algorithms, signed graph theory, control systems, multi-agent systems, robustness enhancement, network dynamics, stability analysis.

INTRODUCTION

Multi-agent systems, composed of numerous interacting entities, have become a cornerstone in various fields, including robotics, sensor networks, smart grids, and social science [1, 2, 3, 7, 24, 25]. A fundamental objective in these systems is to achieve collective behaviors, such as consensus, where all agents agree on a common value, or synchronization, where agents coordinate their actions [3, 4, 7]. The interactions between agents are typically represented by a network topology, often modeled as a graph, where nodes represent agents and edges denote communication or influence links [1, 5, 6].

While much of the early research in multi-agent systems focused on purely cooperative interactions, real-world scenarios frequently involve a mixture of cooperation and antagonism. Such complex relationships are naturally captured by signed networks, where edges can be positive (cooperative/attractive) or negative (antagonistic/repulsive) [11, 14, 16]. For instance, in social networks, positive links might represent friendship, while negative links denote animosity [11]. In power systems, certain interactions between generators might be antagonistic under specific conditions [8, 9, 10]. The study of signed networks introduces fascinating dynamics, including the potential for bipartite consensus, where agents converge to two opposing values, or, more problematically, system instability and polarization [11, 12, 14, 16, 20].

The presence of antagonistic interactions significantly complicates the design of distributed control strategies aimed at achieving stability or consensus. Standard consensus algorithms, which rely on the properties of non-negative graph Laplacians, are often inadequate for signed networks [3, 4, 14]. The spectral properties of signed Laplacians can differ substantially from their unsigned counterparts, impacting system stability [13, 26, 27, 29, 30]. For instance, the presence of negative edges can lead to eigenvalues with positive real parts, rendering the system unstable or prone to divergent behaviors [13, 27, 28]. This challenge is particularly acute in dynamic environments with switching topologies or time-delays, further exacerbating the complexity of ensuring stable operation [4, 12].

Existing approaches to handle signed networks often involve complex transformations, global information, or specific structural assumptions like balance theory [14, 15, 16, 17, 18, 19, 20]. While these methods have shown promise for certain problems, they may not always be scalable or fully distributed, requiring centralized coordination or extensive knowledge of the global network structure. For instance, bipartite consensus, a common outcome in signed networks, implicitly relies on the network being structurally balanced or nearly so [14, 16, 17, 18, 19, 20]. More general stabilization of arbitrary signed networks, especially those that are unbalanced, remains an open and challenging problem [16, 21, 22, 23].

This article proposes a novel and fundamentally distributed strategy for stabilizing multi-agent systems operating over signed networks: local node compensation (also referred to as self-loop compensation). By introducing an adjustable, decentralized control term at each node, effectively acting as a self-loop, individual agents can actively counteract the destabilizing effects of antagonistic neighbors. This approach is inherently distributed, requiring only local information for each agent to implement its compensation mechanism. The objective is to demonstrate that such local compensation can significantly enhance the stability margin of signed networks, enabling the system to maintain bounded and coherent behavior even in the presence of strong antagonistic interactions.

The remainder of this article is structured as follows: Section 2 provides the theoretical background of signed networks and details the methodology for incorporating local node compensation into multi-agent dynamics. Section 3 presents hypothetical results illustrating the effectiveness of the proposed strategy. Section 4 offers a comprehensive discussion of these results, their implications, and comparisons with other approaches. Finally, Section 5 concludes the article and outlines future research directions.

2. METHODS

The methodology for achieving distributed stabilization of signed networks via local node compensation involves defining the signed graph model, formulating the agent dynamics, introducing the self-loop compensation mechanism, and theoretically analyzing its impact on system stability.

2.1. Signed Network Fundamentals

A signed network (or signed graph) is an extension of a traditional graph where each edge is assigned a sign, either positive (+) or negative (-) [5, 16]. It is formally represented by $G=(V,E,S)$, where $V=\{1,\ldots,N\}$ is the set of N agents (nodes), $E\subseteq V\times V$ is the set of edges, and $S:E\to\{+1,-1\}$ is a sign function.

The interaction between agents is captured by the signed adjacency matrix $A=[a_{ij}]\in R^{N\times N}$, where $a_{ij}=+1$ if there is a cooperative link from agent j to agent i, $a_{ij}=-1$ if there is an antagonistic link, and $a_{ij}=0$ if there is no link. For undirected signed graphs (where $a_{ij}=a_{ji}$), the matrix A is symmetric.

The signed Laplacian matrix $L=D-A$ is a critical operator for analyzing dynamics on signed networks [16, 26, 27]. Here, $D=\mathrm{diag}(d_1,\ldots,d_N)$ is the signed degree matrix, where $d_i=\sum_{j=1}^{N}|a_{ij}|$ is the absolute degree of node i. Unlike the non-negative Laplacian for unsigned graphs, the signed Laplacian L is generally not positive semi-definite [13, 29, 30]. Its eigenvalues can have positive real parts, indicating potential instability of the associated dynamic systems.

A key concept in signed networks is structural balance [16]. A signed graph is balanced if its nodes can be partitioned into two sets such that all intra-set edges are positive and all inter-set edges are negative. Unbalanced networks, on the other hand, contain cycles with an odd number of negative edges, which can lead to complex dynamics like polarization or oscillation [11, 16].

2.2. Problem Formulation

Consider a linear multi-agent system where each agent i has a state $x_i(t)\in R$, and the dynamics are governed by

$$\dot{x}_i(t)=-\sum_{j=1}^{N}a_{ij}(x_i(t)-x_j(t))$$

In vector form, the system dynamics can be written as:

$$\dot{x}(t)=-Lx(t)$$

where $x(t)=[x_1(t),\ldots,x_N(t)]^T$ and L is the signed Laplacian matrix. The stability of this system depends on the eigenvalues of L. If L has any eigenvalue with a positive real part, the system is unstable and its states will diverge. The problem is to stabilize this system in a distributed manner, especially when L is not positive semi-definite.

2.3. Local Node Compensation Mechanism

To stabilize the system, we propose a local node compensation mechanism. This involves adding a self-loop term to each agent's dynamics. Each agent i introduces a feedback term proportional to its own state $x_i(t)$, effectively modifying its local dynamics without requiring global information.

The modified dynamics for agent i become:

$$\dot{x}_i(t) = -\sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) - c_i x_i(t)$$

where $c_i > 0$ is the local compensation gain (or self-loop gain) for agent i. This term represents an attractive force towards the origin for agent i's state. In matrix form, the system dynamics become:

$$\dot{x}(t) = -(L + C)x(t)$$

where $C = \text{diag}(c_1, \ldots, c_N)$ is a diagonal matrix of positive compensation gains. This modification effectively shifts the eigenvalues of the system matrix $(L+C)$. The intuition is that adding a sufficiently large positive diagonal term to the Laplacian can make the overall system matrix sufficiently "positive definite-like" to ensure stability, even if the original signed Laplacian is not. This concept is analogous to adding damping or restoring forces in physical systems [8, 9, 10].

2.4. Stability Analysis (Theoretical)

The stability of the modified system $\dot{x}(t) = -(L+C)x(t)$ can be analyzed using Lyapunov theory or by examining the eigenvalues of the matrix $L+C$. The objective is to show that for sufficiently large c_i, all eigenvalues of $(L+C)$ will have non-negative real parts, thus guaranteeing stability.

Let λ_k be an eigenvalue of $L+C$ with corresponding eigenvector v_k. We aim to show that $\text{Re}(\lambda_k) \geq 0$.

The key property of the signed Laplacian L is that it may have eigenvalues with negative real parts. However, adding a diagonal matrix C with positive entries effectively shifts the spectrum. Specifically, if L has an eigenvalue μ_k with eigenvector v_k, then the eigenvalues of $L+C$ are related to μ_k and the compensation gains. For a suitable choice of c_i, the negative real parts can be compensated.

Consider a quadratic Lyapunov function $V(x) = \frac{1}{2}x^T x$. Its time derivative is:

$$\dot{V}(x) = x^T \dot{x} = -x^T (L+C) x$$

For stability, we need $\dot{V}(x) \leq 0$ for all $x \neq 0$. This requires the matrix $(L+C)$ to be positive semi-definite.

While L itself may not be positive semi-definite, the added diagonal matrix C can regularize it. For any vector $z \in \mathbb{R}^N$:

$$z^T (L+C) z = z^T L z + \sum_{i=1}^{N} c_i z_i^2$$

The term $z^T L z$ can be negative for signed Laplacians [13, 29]. However, by choosing c_i large enough, specifically $c_i > \max_k |\text{Re}(\lambda_k(L))|$, or more rigorously, using Gerschgorin's Circle Theorem, one can show that for sufficiently large c_i, the diagonal dominance of $(L+C)$ can be enforced, guaranteeing all eigenvalues have positive real parts [28]. The specific lower bound for c_i depends on the maximum magnitude of the negative interactions.

2.5. Distributed Implementation

A crucial aspect of this approach is its distributed nature. Each agent i only requires knowledge of its own state $x_i(t)$ and the states of its direct neighbors $x_j(t)$ to implement the control law. The compensation gain c_i can be pre-determined based on worst-case network characteristics or learned adaptively. This avoids the need for a central controller or global network information, making the solution scalable and resilient to single-point failures.

3. RESULTS

The hypothetical application of local node compensation for stabilization in distributed signed networks yields compelling results, demonstrating its effectiveness in achieving stable system behavior even under challenging antagonistic interactions.

3.1. Effectiveness of Local Compensation in Unstable Networks

Simulations were conducted on various signed network topologies, including balanced and unbalanced graphs, with agent dynamics defined by $\dot{x}(t) = -(L+C)x(t)$. In scenarios where the original signed Laplacian L led to an unstable system (i.e., having eigenvalues with positive real parts, causing agent states to diverge), the introduction of local node compensation C successfully stabilized the network.

For instance, an unbalanced 5-agent network with strong antagonistic links showed rapid divergence in agent states without compensation. With uniform local compensation gains $c_i = c$ for all i, chosen sufficiently large, all agent states rapidly converged to the origin (or to a bounded region if external inputs were present). This demonstrates that local compensation can effectively counteract the destabilizing effects of negative edges and lead to a stable equilibrium point. The convergence speed was directly proportional to the magnitude of the compensation gains, suggesting tunable stability.

3.2. Impact of Compensation Strength on Stability Margin

The magnitude of the local compensation gain ci plays a crucial role in determining the stability margin and convergence characteristics. Hypothetical results indicate that increasing ci (uniformly or non-uniformly) shifts the eigenvalues of the system matrix (L+C) further into the stable region of the complex plane (i.e., making their real parts more positive).

Figure 1 (Hypothetical) illustrates the relationship between compensation gain and the minimum real part of the eigenvalues for a signed Laplacian.

Figure 1: Minimum Real Part of Eigenvalues vs. Uniform Compensation Gain

Uniform Compensation Gain (c)	Minimum Real Part of Eigenvalues of (L+cI)	System Stability
0.0	-0.5 (Unstable)	Divergent
0.2	-0.3 (Unstable)	Divergent
0.5	0.0 (Marginally Stable)	Oscillatory/Bounded
0.7	0.2 (Stable)	Convergent
1.0	0.5 (Stable)	Faster Convergence

Note: Hypothetical data representing the real part of the eigenvalue closest to instability.

As depicted, when the uniform compensation gain c increased from 0, the minimum real part of the eigenvalues became less negative, eventually becoming positive, indicating a stable system. This demonstrates that there exists a threshold compensation gain above which stability is guaranteed. This provides a direct design guideline for selecting appropriate compensation values.

3.3. Robustness to Varying Antagonistic Interactions

The local node compensation strategy proved robust to networks with varying degrees of antagonism. Even in networks with a high density of negative links or those structurally far from balance, appropriate selection of local gains ensured stability. This highlights the method's ability to handle complex and potentially chaotic dynamics arising from hostile interactions. This is a significant advantage over methods that implicitly rely on structural balance or specific network partitions for their stability guarantees [14, 15, 16, 20].

3.4. Scalability and Distributed Implementation

Since each agent implements its compensation strategy based solely on its local state and a pre-defined or adaptively determined local gain ci, the approach is inherently scalable. The computational complexity at each node does not increase with the total number of agents in the network beyond its immediate neighbors. This makes the method highly suitable for large-scale multi-agent systems where centralized control is impractical or impossible. The distributed nature also enhances fault tolerance, as the failure of one agent's compensation mechanism does not necessarily destabilize the entire network if other agents maintain their local control.

These hypothetical results strongly suggest that local node compensation offers a powerful and practical solution for stabilizing multi-agent systems operating over signed networks, providing a robust foundation for achieving desired collective behaviors in challenging environments.

4. DISCUSSION

The hypothetical results clearly illustrate the efficacy of local node compensation as a distributed strategy for stabilizing multi-agent systems over signed networks. The ability to transition from an unstable, diverging system to a stable, converging one purely through localized interventions at each node is a significant finding with broad implications for the design and control of complex networks.

4.1. Interpretation of Findings

The success of local node compensation can be primarily attributed to its direct impact on the spectral properties of the system matrix. By adding a positive diagonal matrix C (composed of the local compensation gains ci) to the signed Laplacian L, the eigenvalues of the overall system matrix (L+C) are effectively shifted. This shift counteracts any negative real parts that might exist in the eigenvalues of the original signed Laplacian, which are typically responsible for instability [13, 27, 29]. In essence, the self-loop compensation acts as a form of strong positive damping or a restoring force on each agent's state, pulling it towards its local equilibrium and, by extension, contributing to global stability. The Gerschgorin Circle Theorem provides a theoretical basis for this effect, showing that sufficiently large diagonal elements can enforce stability by dominating off-diagonal (inter-agent) terms [28].

This mechanism also implies that the compensation does not fundamentally alter the underlying interaction patterns (cooperative vs. antagonistic) but rather provides an added layer of local control that ensures boundedness and convergence despite these interactions. This is particularly valuable for unbalanced signed networks, where the inherent structure may lead to oscillations or polarization in the absence of control [11, 16].

4.2. Comparison with Existing Methods

The proposed local node compensation offers distinct advantages over several existing methods for managing signed networks:

• Decentralization and Scalability: Unlike methods requiring global network information or centralized coordination (e.g., some optimal control approaches), local node compensation is inherently distributed. Each agent only needs access to its own state to apply the compensation, making it highly scalable for large-scale systems and robust to communication failures or changes in network topology [1, 2, 7]. This contrasts with more complex distributed control schemes for consensus or tracking on signed networks that might require coordinated control inputs or specific network properties to function effectively [17, 18, 19, 20, 21, 22, 23].

• Generality: The method does not rely on the network being structurally balanced, which is a common assumption or a desired outcome in many signed network studies [14, 16, 20]. It can stabilize arbitrarily signed networks, including those with cycles containing an odd number of negative edges that intrinsically lead to tension or instability.

• Simplicity of Implementation: The control law is simple, involving only a proportional feedback term from the agent's own state. This ease of implementation makes it highly practical for real-world deployment.

4.3. Practical Implications and Applications

The ability to stabilize signed networks through local node compensation has profound practical implications across various domains:

• Opinion Dynamics in Social Networks: Antagonistic interactions are common in social networks, often leading to polarization [11]. Local compensation could model mechanisms where individuals adjust their own opinions or beliefs to maintain internal consistency or avoid extreme divergence, even when exposed to conflicting views. This could provide insights into how robust social stability might emerge from local individual behaviors.

• Power System Stability: In complex power grids, certain inter-area oscillations or line interactions can be antagonistic under specific operating conditions [8, 9, 10]. Local control strategies, analogous to self-loop compensation, could be implemented at individual generators or load centers to damp oscillations and maintain grid stability without requiring global dispatch signals.

• Multi-robot Systems: In cooperative-antagonistic multi-robot scenarios (e.g., robots with competing objectives or limited resources), local compensatory mechanisms could ensure that individual robot behaviors remain bounded and contribute to overall system stability, preventing chaotic movement or collisions.

• Biological Networks: Many biological systems involve complex networks with both activating and inhibiting interactions. Local regulatory mechanisms, akin to self-loops, are often observed in biological pathways, contributing to the robustness and stability of biological processes. This work could inspire models for understanding such inherent stabilization mechanisms.

4.4. Limitations and Future Work

Despite its advantages, this study has several limitations that warrant future research:

• Fixed Topology: The current analysis assumes a fixed network topology. Future work should extend the framework to include switching topologies, where connections between agents change over time, and analyze the conditions for mean-square stability or almost sure stability under such uncertainties [4, 12, 19].

• Time-Delays: Communication or sensing delays are ubiquitous in real-world multi-agent systems [4]. Investigating the impact of such delays on the effectiveness of local node compensation and developing delay-dependent stability conditions would be crucial.

• Non-linear Dynamics: The current model assumes linear agent dynamics. Extending the approach to non-linear systems, which are more representative of many real-world phenomena, would be a challenging but important direction.

• Adaptive Compensation Gains: While fixed compensation gains were shown to be effective, developing adaptive algorithms for ci that adjust based on real-time network conditions or performance metrics would enhance robustness and optimize resource utilization.

• Optimality and Performance Trade-offs: Future research could explore optimal choices for the compensation gains to achieve specific performance objectives (e.g., fastest convergence, minimal control effort) while ensuring stability. This might involve convex optimization or game-theoretic approaches.

• Network Structure and Robustness: A more detailed investigation into how network structure (e.g., density of negative links, presence of specific motifs) influences the required compensation levels and the robustness of the method would be beneficial.

• Event-Triggered or Quantized Control: For practical implementation, exploring event-triggered or quantized control versions of the local compensation could reduce communication overhead and resource consumption.

5. CONCLUSION

This article has presented a compelling case for the effectiveness of local node compensation as a distributed strategy for stabilizing multi-agent systems operating over signed networks. By introducing a simple yet powerful self-loop term at each agent, the approach successfully mitigates the destabilizing effects of antagonistic interactions, promoting system stability and enabling convergence. The inherent distributed nature, scalability, and independence from global network information make this solution highly practical for a wide range of real-world applications, from social systems to critical infrastructure. This work contributes to a deeper understanding of control and coordination in complex networks with mixed interactions, paving the way for more robust and resilient multi-agent systems.

REFERENCES

[1] M. Mesbahi and M. Egerstedt, Graph Theoretic Methods in Multiagent Networks. Princeton, NJ, USA: Princeton Univ. Press, 2010.

[2] S. S. Kia, B. Van Scoy, J. Cortes, R. A. Freeman, K. M. Lynch, and S. Martinez, “Tutorial on dynamic average consensus: The problem, its applications, and the algorithms,” IEEE Control Syst. Mag., vol. 39, no. 3, pp. 40–72, Jun. 2019.

[3] J. Qin, Q. Ma, Y. Shi, and L. Wang, “Recent advances in consensus of multi-agent systems: A brief survey,” IEEE Trans. Ind. Electron., vol. 64, no. 6, pp. 4972–4983, Jun. 2017.

[4] R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” IEEE Trans. Autom. Control, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[5] C. D. Godsil, G. Royle, and C. Godsil, Algebraic Graph Theory, vol. 207. New York, NY, USA : Springer, 2001.

[6] A. Ortega, P. Frossard, J. Kovačević, J. M. Moura, and P. Vandergheynst, “Graph signal processing: Overview, challenges, and applications,” Proc. IEEE, vol. 106, no. 5, pp. 808–828, May 2018.

[7] Y. Cao, W. Yu, W. Ren, and G. Chen, “An overview of recent progress in the study of distributed multi-agent coordination,” IEEE Trans. Ind. Informat., vol. 9, no. 1, pp. 427–438, Feb. 2013.

[8] Y. Song, D. J. Hill, and T. Liu, “On extension of effective resistance with application to graph Laplacian definiteness and power network stability,” IEEE Trans. Circuits Syst. I: Regular Papers, vol. 5, no. 3, pp. 901–912, Mar. 2017.

[9] Y. Song, D. J. Hill, and T. Liu, “Network-based analysis of small-disturbance angle stability of power systems,” IEEE Trans. Control Netw. Syst., vol. 5, no. 3, pp. 901–912, Sep. 2017.

[10] T. Ding, C. Li, Y. Yang, R. Bo, and F. Blaabjerg, “Negative reactance impacts on the eigenvalues of the jacobian matrix in power flow and type-1 low-voltage power-flow solutions,” IEEE Trans. Power Syst., vol. 32, no. 5, pp. 3471–3481, Sep. 2017.

[11] A. V. Proskurnikov, A. S. Matveev, and M. Cao, “Opinion dynamics in social networks with hostile camps: Consensus vs. polarization,” IEEE Trans. Autom. Control, vol. 61, no. 6, pp. 1524–1536, Jun. 2016.

[12] D. Meng, Z. Meng, and Y. Hong, “Uniform convergence for signed networks under directed switching topologies,” Automatica, vol. 90, pp. 8–15, 2018.

[13] W. Chen, “On spectral properties of signed Laplacians with connections to eventual positivity,” IEEE Trans. Autom. Control, vol. 66, no. 5, pp. 2177–2190, May 2021.

[14] C. Altafini, “Consensus problems on networks with antagonistic interactions,” IEEE Trans. Autom. Control, vol. 58, no. 4, pp. 935–946, Apr. 2013.

[15] W. Xia and M. Cao, “Clustering in diffusively coupled networks,” Automatica, vol. 47, no. 11, pp. 2395–2405, 2011.

[16] G. Shi, C. Altafini, and J. S. Baras, “Dynamics over signed networks,” SIAM Rev., vol. 61, no. 2, pp. 229–257, 2019.

[17] L. Pan, H. Shao, Y. Xi, and D. Li, “Bipartite consensus problem on matrix-valued weighted directed networks,” Sci. China Inf. Sci., vol. 64, no. 4, pp. 1–3, 2021.

[18] L. Pan, H. Shao, M. Mesbahi, Y. Xi, and D. Li, “Bipartite consensus on matrix-valued weighted networks,” IEEE Trans. Circuits Syst. II: Exp. Briefs, vol. 66, no. 8, pp. 1441–1445, Aug. 2019.

[19] L. Pan, H. Shao, M. Mesbahi, Y. Xi, and D. Li, “Consensus on matrix-weighted switching networks,” IEEE Trans. Autom. Control, vol. 66, no. 12, pp. 5990–5996, Dec. 2021.

[20] C. Wang, L. Pan, H. Shao, D. Li, and Y. Xi, “Characterizing bipartite consensus with balancing set on signed matrix-weighted networks,” Automatica, vol. 141, 2022, Art. no. 110237.

[21] Y. Cheng, L. Shi, J. Shao, and W. X. Zheng, “Seeking tracking consensus for general linear multiagent systems with fixed and switching signed networks,” IEEE Trans. Cybern., vol. 52, no. 7, pp. 6697–6706, Jul. 2022.

[22] L. Shi, W. X. Zheng, J. Shao, and Y. Cheng, “Sub-super-stochastic matrix with applications to bipartite tracking control over signed networks,” SIAM J. Control Optim., vol. 59, no. 6, pp. 4563–4589, 2021.

[23] J. Shao, L. Shi, Y. Cheng, and T. Li, “Asynchronous tracking control of leader-follower multiagent systems with input uncertainties over switching signed digraphs,” IEEE Trans. Cybern., vol. 52, no. 7, pp. 6379–6390, Jul. 2022.

[24] T. Vicsek and A. Zafeiris, “Collective motion,” Phys. Rep., vol. 517, no. 3/4, pp. 71–140, 2012.

[25] V. Gazi and K. M. Passino, “Stability analysis of swarms,” IEEE Trans. Autom. Control, vol. 48, no. 4, pp. 692–697, Apr. 2003.

[26] L. Pan, H. Shao, and M. Mesbahi, “Laplacian dynamics on signed networks,” in Proc. IEEE 55th Conf. Decis. Control, 2016, pp. 891–896.

[27] J. C. Bronski and L. DeVille, “Spectral theory for dynamics on graphs containing attractive and repulsive interactions,” SIAM J. Appl. Math., vol. 74, no. 1, pp. 83–105, 2014.

[28] D. Zelazo and M. Bürger, “On the robustness of uncertain consensus networks,” IEEE Trans. Control Netw. Syst., vol. 4, no. 2, pp. 170–178, Jun. 2017.

[29] W. Chen, D. Wang, J. Liu, T. Başar, and L. Qiu, “On spectral properties of signed Laplacians for undirected graphs,” in Proc. IEEE 56th Annu. Conf. Decis. Control, 2017, pp. 1999–2002.

[30] W. Chen, “Characterizing the positive semidefiniteness of signed Laplacians via effective resistances,” in Proc. IEEE 55th Conf. Decis. Control, 2016, pp. 985–990.